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ON THE THEORY OF OCEAN CURRENTS DRIVEN BY WINDS IN AN ANISOTROPIC OCEAN

(Part 1)

Yasukazu SAITO

(Received on Dec. 15, 1951)

Contents

Chapter 1. Stationary state of drift currents in a rotating canal (The case of finite depth).

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Appendix: Short note on the theory of drift currents driven by wind in an enclosed infinitely deep ocean (The case of the variation of the Coriolis parameter with latitude).

Chapter 3. Transient state of drift currents in a rotating canal (The case of finite depth).

Summary

One of the modern questions in dynamical oceanography is the role played by the lateral mixing in the movement of sea waters. Among the various researches in this direction, the influence of lateral mixing on the wind-driven currents will be one of the most important.

Recently the problems of wind-driven circulation in enclosed oceans have been solved by several authors, the use being made of the mass transport and taking the effects of the earth's rotation, lateral and vertical mixing into account.

The mass transport method, however, does not permit us to peep at the vertical structure of the oceanic currents. In this respect it is desireable to treat these problems by using the velocity components themselves instead of using the mass transport. The present paper is the result of the efforts in this direction.

This study is made of the wind-driven currents in a homogeneous ocean under the influence of surface wind stress, lateral and vertical mixing, bottom friction (or no bottom friction), horizontal pressure gradient caused by a variable surface height, and Coriolis force.

In Chapter 1 of this work is given the analytical solution of the stationary state of drift currents in a rotating canal of finite depth. In 1942, Dr. Okamoto discussed a case of the drift currents in an anisotropic ocean under the assumtion that the wind sweeps over a finite area of the sea surface. But, since in his case the sea is not bounded by lands, his treatment is not exact, because $S_x=0$ and $S_y=0$, where S_x and S_y mean the total flow, are used simultaneously as the conditions of continuity.

Thus, in the present paper, the author tries to obtain an analytical solution for the problem in a rotating canal, taking account of the terms of the horizontal kinematic eddy viscosity $\nu_x \frac{\partial^2 W}{\partial x^2}$ and $\nu_y \frac{\partial^2 W}{\partial y^2}$, and that of the vertical kinematic eddy viscosity $\nu_z \frac{\partial^2 W}{\partial z^2}$ (where W means the velocity complex vector).

In Chapter 2, the author gives the theory of the transient state of drift currents in infinite depth. As to the theory of drift currents in a transient state, it was developed by several authors. But all these attempts are confirmed to the case of the drift currents discussed by taking account of the vertical Austausch only as a frictional term.

So, in the present chapter the author mathematically makes clear the properties of the drift currents in a transient state and discusses in what manner the drift currents gradually develop in an ocean initially at rest and how they gain a stationary state, taking the effect of the horizontal and vertical Austausch into account simultaneously.

For the purpose of avoiding the tediousness of mathematical treatment, however, we deal with the following two special cases:

(i) A case of the drift currents in an infinitely deep ocean, when the values of the coeffeients of the horizontal and vertical kinematic eddy viscosity differ to each other and the ocean is boundless in horizontal extent and initially at rest, under the circumstance that the constant wind suddenly begins to blow over only a finite area of the sea surface.

(ii) A case of the drift currents in an infinitely deep canal, where the conditions used are of the same ones as (i).

Moreover, in Appendix, short note on the theory of drift currents driven by wind in an enclosed infinitely deep ocean is published. And we treat here the above problem, taking account of the variation of the Coriolis parameter with latitude. But as the mathematical treatment in above case is very tedious, the detail of this problem is reserved for the next paper.

In Chpater 3 we aim to deal with the case of the transient state of drift currents in a rotating canal of finite depth. As to the theory of this case, it has been already propounded by Horrock (1927) and Prot. T. Nomitsu

(1934). But in their treatments the terms of the horizontal mixing in the equation of motion were entirely neglected. Therefore, the horizontal current velocity obtained by them does not vanish at the coast-lines themselves, provided the wind stress is not zero along these ones. To avoid this defect, introducing the terms of the horizontal and vertical mixing into the equation of motion used, we treat the case of the transient state of current and surface slope generated by wind action. (Where, in our case the variation of the Coriolis parameter with latitude is neglected. The detail of the treatment in this case will be reserved for the near future.)

Chapter 1

STATIONARY STATE OF DRIFT CURRENTS IN A ROTATING CANAL[1]

(The case of finite depth)

§ 1 Introduction

Since the Ekman's (1905) theory of the wind-drift and-slope currents, many papers have been published by H. Jeffreys (1921), Proudman and Doodson (1923), Horrocks (1927), J. E. Fjeldstad (1930), Prof. K. Hidaka (1933), Prof. T. Nomitsu (1933), and others. Especially, the theory of the stationary and transient state of drift and slope currents in a canal limited by two lands was discussed by Prof. T. Nomitsu (1934) and Dr. K. Koenuma (1934). In all the above works, however, the vertical Austausch only is taken into consideration as a frictional term.

In 1936, an attempt to introduce the terms of lateral mixing into the hydrodynamical equation of ocean currents as a frictional term were firstly worked out by C. G. Rossby. By means of this conception, K. Hidaka (1939) solved this problem[2] in a series of papers by numerical calculations, applying the equations of motion including the terms of the vertical and horizontal Austausch together with the equation of continuity containing the vertical current.

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1) The outline of this results was announced at the November, 1948 and the September, 1949 meeting of the Oceanographical Society of Japan, and it was simply printed in Japanese in the Journal of the Meteorological Society of Japan (Vol 28, No. 2, 1950).

2) Recently, J. E. Fjeldstad (1946) discussed a special case of drift currents in a heterogeneous water, applying the equation of motion including the terms of the vertical and horizontal Austausch $\left(\nu_x \frac{\partial^2 u}{\partial y^2} \text{ and } \nu_z \frac{\partial^2 u}{\partial z^2}\right)$.

Subsequently, in 1942 Dr. Okamoto discussed a case of the drift currents in an anisotropic ocean[3] in which the coefficient of horizontal kinemetic eddy viscosity ν_x is different from that of vertical kinematic eddy viscosity ν_z, under the assumption that the wind sweeps over only a finite area of the sea surface from $x=0$ to $x=l$.

Nevertheless, in his case, the sea is boundless in horizontal extent and not bounded by lands, $S_x=\int_{-h}^{0} u dz=0$ and $S_y=\int_{-h}^{0} v dz=0$ are used simultaneously as the condition of continuity, where (S_x, S_y) are the total flow, (u, v) the current velocity, h the depth of the sea. And, by applying this formula, he dealt with the ocean currents in the neighbourhood of the north-east trade wind zone in the north hemisphere.

But it is doubtful whether we can get $S_y=0$, or not, though $S_x=0$ is exact in this case because the wind area is bounded in the direction of x-axis (where x-and y-axes are taken positive northward and westward respectively). If the y-axis is bounded by lands, the above formula ($S_y=0$) is right, but we can not use this formula in the former case. I suppose that he assumed $S_y=0$ to determine γ_y definitely, because two unknown quantities (γ_x, γ_y) were involved in the hydrodynamical equations, where quantities (γ_x, γ_y) are the gradient of surface elevation, and he evaluated γ_y by using the method mentioned above. But it appears to be unallowable since in that case the y-axis must be at least bounded by a land.

Therefore, in the present paper, the author deals with the case of a canal, assuming that the y-axis is limited by both lands ($y=0$, $y=a$; a the width of a canal) and that the wind sweeps over only a finite area of the surface ($y=0$, $y=a$, $0<x<l$; l the breadth of a wind zone).

Thus, he tries to obtain an analytical solution for the problem in a rotating canal, taking account of the terms of the horizontal kinematic eddy viscosity $\nu_x \frac{\partial^2 W}{\partial x^2}$ and $\nu_y \frac{\partial^2 W}{\partial y^2}$*, and that of the vertical kinematic eddy viscosity $\nu_z \frac{\partial^2 W}{\partial z^2}$ (where $W=u+iv$, $i=\sqrt{-1}$; W the velocity complex vector), and using the previous two equations ($S_x=0$ and $S_y=0$) as the condition of continuity. It seems that his treatment under the above circumstances may be adequate to the case where the north-east trade wind sweeps over the sea surface between two lands.

---

3) The theory of drift currents in an anisotropic ocean has firstly been discussed by H. Ertel (1939) and T. Nomitsu, neglecting the term of the horizontal Austausch after Ekman.

* We can not neglect the term of the horizontal kinematic eddy viscosity $\nu_y \frac{\partial^2 W}{\partial y^2}$ in the hydrodynamical equation, because there appears two degrees freedom from the second derivative with respect to y, and they supply two boundary conditions for W.

In this case, however, he could not expect the theoretical results of the westward intensification** firstly propounded by Stommel (1948), Munk (1950), and K. Hidaka (1951), the use being made of the "integrated mass transport***", because he assumed the Coriolis parameter as a constant in a similar manner as in the Stockman's paper (1946).

Though the mass transport method enables us to reduce the problems to two dimensional ones, and makes it easy to treat them mathematically, it does not permit us to peep at the vertical structure of the ocean currents. But it is able to treat these problems by using the velocity components themselvs instead of using the integrated mass transport, taking account of the three mixing terms.

It may, furthermore, be said that even if the Coriolis parameter is assumed as a constant, the results taken under this assumption are plausible, as long as the region under considration is not far from the wind zone of a small scale. From the point of view mentioned above, in § 6 the results obtained by the present author may be disussed in comparison to Munk's ones.

And the case of the variation of the Coriolis parametes with latitude will be discussed in the near future (Part 2).

§ 2. The equation of motion and boundary condition

First, consider a rotating canal of uniform depth bounded by two infinite parallel straight coasts distance a apart and suppose that a constant (but not necessarilly uniform) wind sweeps over only a finite area of a canal surface from $x=0$ to $x=l$ (see Fig. 1). Choose the co-ordinate axes such that the z-axis is directed vertically upwards from a level surface (assumed plane) and the x- and y-axes lie on the undisturbed sea surface (where the x-axis parallel to the coasts, the y-axis perpendiculary to the coasts. Let the following notations be used:

T = the tractive force of the wind per unit area of the sea surface, and it is uniform for $0<x<l$, and vanishes for $x>l$ and $x<0$,

ζ = the elevation of the free surface from the undisturbed level,

γ_x, γ_y = the component slope of the sea surface in the direction of the x-and y-co-ordinate axes respectively (where $\gamma_x=\frac{\partial\zeta}{\partial x}$, $\gamma_y=\frac{\partial\zeta}{\partial y}$),

---

** In 1948, Dr. Stommel showed that the westward intensification of stream line in the wind-driven oceanic circulation was a result of the variation of the Coriolis parameter with latitude.

*** $M_x=\int_{-h}^{0}\rho u dz$, $M_y=\int_{-h}^{0}\rho v dz$ (where M_x and M_y are the integrated mass transport, ρ the density of sea waters).

ϕ = the angle of T counter clockwise from the positive direction of the x co-ordinath axis,

h = the depth of a canal,

ρ = the density of the sea water,

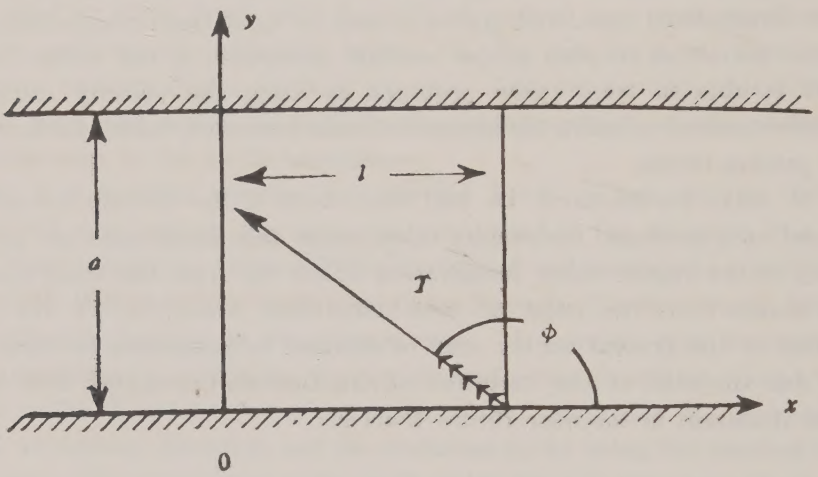

Fig. 1

μ = the vertical coefficient of eddy viscosity of the water,

ν_x, ν_y, ν_z = the coefficient of kinematic eddy viscosity of the water in the direction of x, y, and z co-ordinate axes respectively (where, $\nu_z = \frac{\mu}{\rho}$),

u, v = the component velocity of currents in the direction of x and y co-ordinate axes respectively,

ω = the angular velocity of the earth's rotation,

φ = the latitude of the place under consideration,

$\bar{\omega} = \omega \sin \varphi$,

g = the accelation of gravity,

f = the coefficient of friction at the bottom, assuming the resistance proportional to the slip velocity,

S_x, S_y = the total flow in the direction of x-and y-co-ordinate axes respectively.

If the non-linear enertial terms and the vertical velocity in the hydrodynamical equations may be considered neglisible, then the currents and the surface slope produced by wind, in a stationary state, will be represented by the solution of the following equations:

$$\left.\begin{aligned} \nu_x\frac{\partial^2 u}{\partial x^2}+\nu_y\frac{\partial^2 u}{\partial y^2}+\nu_z\frac{\partial^2 u}{\partial z^2}-2\bar{\omega}v-g\gamma_x=0 \\ \nu_x\frac{\partial^2 v}{\partial x^2}+\nu_y\frac{\partial^2 v}{\partial y^2}+\nu_z\frac{\partial^2 v}{\partial z^2}+2\bar{\omega}u-g\gamma_y=0 \end{aligned}\right\} \quad \cdots\cdots(1)$$

with the equation of continuity

$$S_x=\int_{-h}^{0}u\,dz=0, \qquad S_y=\int_{-h}^{0}v\,dz=0 \quad \cdots\cdots(2)$$

and under the boundary conditions

$$|u|_{y=0}=0, \quad |u|_{y=a}=0, \quad |v|_{y=0}=0, \quad |v|_{y=a}=0 \quad \text{at the coast } (y=0,\ y=a),$$

$$\cdots\cdots(3)$$

$$\mu\left|\frac{\partial u}{\partial z}\right|_{z=0}=T\cos\phi, \quad \mu\left|\frac{\partial v}{\partial z}\right|_{z=0}=T\sin\phi \text{ at the sea surface } (z=0) \text{ for } 0\leqq x\leqq l$$

$$=0 \qquad\qquad =0 \qquad \text{at the sea surface } (z=0) \text{ for } x>l \text{ and } x<0, \quad \cdots\cdots(4)$$

$$\left|\frac{\partial u}{\partial z}-\varkappa u\right|_{z=-h}=0, \qquad \left|\frac{\partial v}{\partial z}-\varkappa v\right|_{z=-h}=0 \quad \text{at the sea bottom } (z=-h) \text{ for } -\infty<x<\infty,$$

$$\cdots\cdots(5)$$

where $\varkappa=\dfrac{f}{\mu}$: positive number.

If we introduce the complex variables: $W=u+iv$, $\gamma=\gamma_x+i\gamma_y$, and $S=S_x+iS_y$,
(where i=imaginary number)

then the above equations and boundary conditions become

$$\nu_x\frac{\partial^2 W}{\partial x^2}+\nu_y\frac{\partial^2 W}{\partial y^2}+\nu_z\frac{\partial^2 W}{\partial z^2}-i2\bar{\omega}W-g\gamma=0 \quad \cdots\cdots(1')$$

$$S=\int_{-h}^{0}W\,dz=0 \quad \cdots\cdots(2')$$

$$|W|_{y=0}=0, \quad |W|_{y=a}=0 \quad \text{at } y=0 \text{ and } y=a \quad \cdots\cdots(3')$$

$$\left.\begin{aligned} \mu\left|\frac{\partial W}{\partial z}\right|_{z=0}&=T\exp\{i\phi\} && (0\leqq x\leqq l) \\ &\text{at } z=0 \\ &=0 && (x<0,\ l<x) \end{aligned}\right\} \quad \cdots\cdots(4')$$

$$\left|\frac{\partial W}{\partial z}-\varkappa W\right|_{z=-h}=0 \quad \text{at} \quad z=-h \quad (-\infty<x<\infty). \qquad \cdots\cdots(5')$$

Moreover, it must be satisfied that W, $\frac{\partial W}{\partial x}$, etc. are continuous at the ends of the wind region $(x=0,\ x=l)$ $\cdots\cdots(6)$

and they are zero at $x=\pm\infty$. $\cdots\cdots(7)$

Therefore, the required current velocity W will be given by the solution of equation (1') that satisfies the equation of continuity (2') and the boundary conditions (3'), (4'), (5'), (6), (7).

§ 3. The solution of diffential equation

(The case of finite bottom-friction depending on the slip velocity)

In order to solve the equation (1'), we shall use Stokes method. Let W be expressed by Fourier's integral, then

$$W=\frac{1}{\pi}\int_0^\infty d\alpha\int_{-\infty}^{\infty} W(\lambda,\ y,\ z)\cos\alpha(x-\lambda)\,d\lambda$$

Next, express $W(\lambda,\ y,\ z)$ with Fourier's sine series by considering the condition (3'), then

$$W(\lambda,\ y,\ z)=\frac{2}{a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^a W(\lambda,\ \beta,\ z)\sin\frac{(2n+1)\pi}{a}\beta\,d\beta.$$

Therefore,

$$W=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^\infty d\alpha\int_{-\infty}^{\infty}\int_0^\infty W(\lambda,\beta,z)\sin\frac{(2n+1)\pi}{a}\beta\cos\alpha(\lambda-x)\,d\beta d\lambda.$$

By putting

$$V_n=\int_0^\infty\int_0^a W(\lambda,\ \beta,\ z)\sin\frac{(2n+1)\pi}{a}\beta\cos\alpha(\lambda-x)\,d\beta d\lambda, \qquad \cdots\cdots(8)$$

we can write W as the following form:

$$W=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^\infty V_n\,d\alpha, \qquad \cdots\cdots(9)$$

Moreover,

$$\frac{\partial^2 W}{\partial x^2}=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}d\alpha\int_{-\infty}^{\infty}\frac{\partial^2 W}{\partial\lambda^2}\sin\frac{(2n+1)\pi}{a}\beta\cos\alpha(\lambda-x)\,d\beta d\lambda.$$

But since

$$\int_{-\infty}^{\infty}\frac{\partial^2 W}{\partial\lambda^2}\cos\alpha(\lambda-x)d\lambda=\left\{\left|\frac{\partial W}{\partial\lambda}\cos\alpha(\lambda-x)\right|_{\lambda=-\infty}^{0}+\left|\frac{\partial W}{\partial\lambda}\cos\alpha(\lambda-x)\right|_{\lambda=0}^{l}\right.$$

$$\left.+\left|\frac{\partial W}{\partial\lambda}\cos\alpha(\lambda-x)\right|_{\lambda=l}^{\infty}\right\}+\left\{\left|\alpha W(\lambda)\sin\alpha(\lambda-x)\right|_{\lambda=-\infty}^{0}+\left|\alpha W(\lambda)\sin\alpha(\lambda-x)\right|_{\lambda=0}^{l}\right.$$

$$\left.+\left|\alpha W(\lambda)\sin\alpha(\lambda-x)\right|_{\lambda=l}^{\infty}\right\}-\alpha^2\int_{-\infty}^{\infty}W(\lambda)\cos\alpha(\lambda-x)\,d\lambda,$$

the values of middle blackets in the above expression become respectively zero by the help of the conditions (6) and (7).
Then we obtain

$$\frac{\partial^2 W}{\partial x^2}=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}(-\alpha^2)\,V_n\,d\alpha. \quad\cdots\cdots(10)$$

And $\frac{\partial^2 W}{\partial y^2}$ also may be dealt with in the same way entirely as that used for $\frac{\partial^2 W}{\partial x^2}$.

Namely, $$\frac{\partial^2 W}{\partial y^2}=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}\left\{-\frac{(2n+1)^2\pi^2}{a^2}\right\}V_n\,d\alpha.$$

(applying (3′))

And moreover, $$\frac{\partial^2 W}{\partial z^2}=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}\frac{\partial^2 V_n}{\partial z^2}d\alpha,$$

$$\frac{\partial W}{\partial z}=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}\frac{\partial V_n}{\partial z}d\alpha. \quad\cdots\cdots(12),\ (13)$$

Next, we may expand $\gamma(x, y)$ and T in Fourier's sine series and integral such as:

$$\gamma(x, y)=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}d\alpha\int_{-\infty}^{\infty}\int_0^{\infty}\gamma(\lambda,\beta)\sin\frac{(2n+1)\pi}{a}\beta\cos\alpha(\lambda-x)\,d\beta d\lambda$$

$$=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}\Gamma_n(\alpha)\,d\alpha, \quad\cdots\cdots(14)$$

where $\Gamma_n=\int_{-\infty}^{\infty}\int_0^a \gamma(\lambda,\beta)\sin\frac{(2n+1)\pi}{a}\beta\cos\alpha(\lambda-x)\,d\beta\,d\lambda$. ..... (15)

And also,

$$T=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}d\alpha\int_{-\infty}^{\infty}\int_0^a T\sin\frac{(2n+1)\pi}{a}\beta\cos\alpha(\lambda-x)\,d\beta d\lambda$$

$$=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}T_n(\alpha)\,d\alpha, \quad \cdots\cdots (16)$$

where $T_n(\alpha)=\int_{-\infty}^{\infty}\int_0^a T\sin\frac{(2n+1)\pi}{a}\beta\cos\alpha(\lambda-x)\,d\beta d\lambda$

$$=\int_0^l\int_0^a T\sin\frac{(2n+1)\pi}{a}\beta\cos\alpha(\lambda-x)\,d\beta d\lambda$$

$$=\frac{2aT}{(2n+1)\pi}\left[\frac{\sin\alpha x}{\alpha}+\frac{\sin\alpha(l-x)}{\alpha}\right]. \quad \cdots\cdots (17)$$

Hence, substitute the above values of (9), (10), (11), (12), and (14) into the equation (1′), and equate the factor of $\sin\frac{(2n+1)\pi}{a}y$,

then $$-\nu_x\alpha^2V_n-\frac{(2n+1)^2\pi^2}{a^2}\nu_yV_n+\nu_z\frac{\partial^2V_n}{\partial z^2}-i2\bar{\omega}V_n-\Gamma_n(\alpha)g=0,$$

namely $$\frac{\partial^2V_n}{\partial z^2}-\left\{\frac{\nu_x}{\nu_z}\alpha^2+\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i2\frac{\bar{\omega}}{\nu_z}\right\}V_n=\frac{\Gamma_n(\alpha)g}{\nu_z}. \quad \cdots\cdots (18)$$

Corresponding to (18), the equation of continuity (2′) becomes as follows: $$\int_{-h}^{0}V_n dz=0. \quad \cdots\cdots (19)$$

Now, by putting $$\frac{\nu_x}{\nu_z}\alpha^2+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}=(p_n+iq_n)^2, \quad \cdots\cdots (20)$$

(where $p_n>q_n>0$)

we have $$\frac{\nu_x}{\nu_z}\alpha^2+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}=p_n^2-q_n^2,\ \frac{\bar{\omega}}{\nu_z}=p_nq_n. \quad \cdots\cdots (21)$$

From the above two equations, we have

$$\left.\begin{aligned} p_n&=\left[\left\{\left(\frac{\nu_x}{2\nu_z}\alpha^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right)^2+\left(\frac{\varpi}{\nu_z}\right)^2\right\}^{\frac{1}{2}}+\left\{\frac{\nu_x}{2\nu_z}\alpha^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right\}\right]^{\frac{1}{2}}\\ q_n&=\left[\left\{\left(\frac{\nu_x}{2\nu_z}\alpha^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right)^2+\left(\frac{\varpi}{\nu_z}\right)^2\right\}^{\frac{1}{2}}-\left\{\frac{\nu_x}{2\nu_z}\alpha^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right\}\right]^{\frac{1}{2}}\end{aligned}\right\}, \quad \cdots\cdots(22)$$

therefore, the equation (18) is as follows:

$$\frac{\partial^2 V_n}{\partial z^2}-(p_n+iq_n)^2V_n=\Gamma_n(\alpha)\,g/\nu_z\,. \quad \cdots\cdots(23)$$

From (4) and (5′), the surface and bottom conditions are as follows:

$$\left.\begin{aligned} \mu\left|\frac{\partial V_n}{\partial z}\right|_{z=0}&=T_n(\alpha)\,exp\{i\phi\} && (0\leqq x\leqq l)\\ &=0 \quad \text{at} \quad z=0 && (x<0,\ x<l)\end{aligned}\right\} \quad \cdots\cdots(24)$$

$$\left|\frac{\partial V_n}{\partial z}-\varkappa V_n\right|_{z=-h}=0 \qquad \text{at} \quad z=-h \quad (-\infty<x<\infty). \quad \cdots\cdots(25)$$

The general solution of (23) is obviously given by

$$V_n=\frac{1}{(p_n+iq_n)^2}\left[A_1\cosh(p_n+iq_n)(h+z)+A_2\sinh(p_n+iq_n)(h+z)-\frac{\Gamma_n g}{\nu_z}\right]$$

(where A_1 and A_2 are the arbitrary constant).

Here, A_1, A_2, $\Gamma_n(\alpha)$, and $V_n(\alpha)$ are determined by (24), (25), and (19). Now by putting $\varkappa h=K$, we can get the following formulae:

$$\begin{aligned}\Gamma_n(\alpha)=&\frac{2a}{(2n+1)\pi}\frac{T}{\rho gh}\,exp\{i\phi\}\\ &\times\frac{(p_n+iq_n)^2h^2\sinh(p_n+iq_n)h+K(p_n+iq_n)h\{\cosh(p_n+iq_n)h-1\}}{\{(p_n+iq_n)^2h^2-K\}\sinh(p_n+iq_n)h+K(p_n+iq_n)h\cosh(p_n+iq_n)h}\\ &\times\left[\frac{\sin\alpha x}{\alpha}+\frac{\sin\alpha(l-x)}{\alpha}\right] \quad \cdots\cdots(26)\end{aligned}$$

$$V_n(\alpha)=\frac{2aT}{\mu(2n+1)\pi}\,exp\{i\phi\}$$

$$\times \frac{\sin\alpha x+\sin\alpha(l-x)}{\alpha(p_n+iq_n)[\{(p_n+iq_n)^2h^2-K\}\sinh(p_n+iq_n)h+K(p_n+iq_n)h\cosh(p_n+iq_n)h]}$$

$$\times [\{(p_n+iq_n)^2h^2+K(\cosh(p_n+iq_n)h-1)\}\cosh(p_n+iq_n)(h+z)+K\{(p_n+iq_n)h$$

$$-\sinh(p_n+iq_n)h\}\sinh(p_n+iq_n)(h+z)-\{(p_n+iq_n)h\sinh(p_n+iq_n)h$$

$$+K(\cosh(p_n+iq_n)h-1)\}]. \quad \cdots\cdots(27)$$

Therefore, the substitution of (26) and (27) into (14) and (9) gives the values of current velocity W and surface slope γ, where

$$\gamma(x,\ y)=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}\Gamma_n(\alpha)\,d\alpha \quad \cdots\cdots(14)$$

$$W(x,\ y,\ z)=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}V_n d\alpha. \quad \cdots\cdots(9)$$

Now, as we must evaluate the following integrals:

$$\int_0^{\infty}\Gamma_n(\alpha)\,d\alpha \qquad \text{and} \qquad \int_0^{\infty}V_n(\alpha)\,d\alpha,$$

let us prepare for the evaluation of the above infinite integrals.

Preparation for the evaluation of the infinite integrals

(A): The determination of the roots of the denominators of V_n and Γ_n. Namely, the determination of the roots of the equation $\psi(z)$, where

$$\psi(z)=[(p_n+iq_n)^2h^2-K]\sinh(p_n+iq_n)h+K(p_n+iq_n)h\cosh(p_n+iq_n)h.$$

$$\cdots\cdots(28)$$

Now, by putting $z_{n,s}=\pm(-b'_{n,s}+ia'_{n,s})$, where $z_{n,s}$ are the roots (with $a'_{n,s}>b'_{n,s}>0$) of the equation $\psi(z)=0$, we can obtain the following

formula: $(p_{n,s}+iq_{n,s})^2=p_{n,s}^2-q_{n,s}^2+i\,2p_{n,s}q_{n,s}=\frac{\nu_x}{\nu_z}z_{n,s}^2+\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\,2\frac{\bar{\omega}}{\nu_z}$

$$=\frac{\nu_x}{\nu_z}\left[(b'^2_{n,s}-a'^2_{n,s})-i2a'_{n,s}b'_{n,s}\right]+\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\,2\frac{\bar{\omega}}{\nu_z}.$$

While, it is obviously seen that the equation $\psi(z)=0$ is satisfied by the values of above z, when $\beta_{n,s}$ are the roots of the equation $\tan\beta_{n,s}=\frac{K\beta_{n,s}}{K+\beta_{n,s}^2}$ which involves an infinite number of real root. And moreover the following equalities: $(p_{n,s}+iq_{n,s})=\pm i\beta_{n,s}$ are settled.
Consequently, we obtain the following formula:

$$\frac{\nu_x}{\nu_z}\left[\left(b'^2_{n,s}-a'^2_{n,s}\right)-i\,2a'_{n,s}b'_{n,s}\right]+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i\,2\,\frac{\bar{\omega}}{\nu_z}=-\frac{\beta_{n,s}^2}{h^2},$$

therefore

$$a'^2_{n,s}-b'^2_{n,s}=\frac{\nu_z}{\nu_x}+\frac{\beta_{n,s}^2}{h^2}+\frac{(2n+1)^2\pi^2}{a_2}\frac{\nu_y}{\nu_x},\qquad a'_{n,s}b'_{n,s}=\frac{\bar{\omega}}{\nu_x}. \quad \cdots\cdots (29)$$

From the equation (29), we have

$$\left.\begin{aligned} a'_{n,s}&=\left[\left\{\left(\frac{\bar{\omega}}{\nu_x}\right)^2+\left(\frac{1}{2}\frac{\nu_z}{\nu_x}\frac{\beta_{n,s}^2}{h^2}+\frac{1}{2}\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_x}\right)^2\right\}^{\frac{1}{2}}+\left(\frac{1}{2}\frac{\nu_z}{\nu_x}\frac{\beta_{n,s}^2}{h^2}\right.\right.\\ &\qquad\left.\left.+\frac{1}{2}\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_x}\right)\right]^{\frac{1}{2}}\\ b'_{n,s}&=\left[\left\{\left(\frac{\bar{\omega}}{\nu_x}\right)^2+\left(\frac{1}{2}\frac{\nu_z}{\nu_x}\frac{\beta_{n,s}^2}{h^2}+\frac{1}{2}\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_x}\right)^2\right\}^{\frac{1}{2}}-\left(\frac{1}{2}\frac{\nu_z}{\nu_x}\frac{\beta_{n,s}^2}{h^2}\right.\right.\\ &\qquad\left.\left.+\frac{1}{2}\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_x}\right)\right]^{\frac{1}{2}}\end{aligned}\right\}. \quad \cdots\cdots (30)$$

If we introduce three quantities D_x, D_y, and D_z such that

$$\sqrt{\frac{\bar{\omega}}{\nu_x}}=\frac{\pi}{D_x},\ \sqrt{\frac{\bar{\omega}}{\nu_y}}=\frac{\pi}{D_y},\quad\text{and}\quad \sqrt{\frac{\bar{\omega}}{\nu_z}}=\frac{\pi}{D_z},$$

the formula (30) becomes

$$\left.\begin{aligned} a'_{n,s}&=\frac{\pi}{D_x}\left[\left\{1+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)^2\right\}^{\frac{1}{2}}+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)\right]^{\frac{1}{2}}\\ b'_{n,s}&=\frac{\pi}{D_x}\left[\left\{1+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)^2\right\}^{\frac{1}{2}}-\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)\right]^{\frac{1}{2}},\end{aligned}\right. \quad \cdots\cdots (31)$$

where the quantity D_z is, of course, the so-called Ekman's "depth of frictional influence", and the other quantities D_x and D_y are a measure for the horizontal turbulence having a dimension of length and very similar to Ekman's "depth of frictional influence" which is the measure for the vertical turbulence. Of course, D_x and D_y are conveniently used in place of ν_x and ν_y themselves, and may be tentatively called the "distance of frictional influence." And moreover, for convenience, let us put $a'_{n,s}=\frac{\pi}{D_x}a_{n,s}$ and $b'_{n,s}=\frac{\pi}{D_x}b_{n,s}$,

where

$$a_{n,s}=\left[\left\{1+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)^2\right\}^{\frac{1}{2}}+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)\right]^{\frac{1}{2}}$$

$$b_{n,s}=\left[\left\{1+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)^2\right\}^{\frac{1}{2}}-\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)\right]^{\frac{1}{2}}.$$

.....(32)

(B): The evaluation of $\int_0^\infty \frac{\varphi(x)}{\psi(x)}\frac{\sin ax}{x}dx$,

where $a>0$, $\lim_{z\to\infty}\left|\frac{\varphi(z)}{\psi(z)}\right|=1$, and $\psi(z)=$ the formula (28).

The roots of the equation $\psi(z)=0$ are $z_{n,s}=-b'_{n,s}+a'_{n,s}$, where $a'_{n,s}>b'_{n,s}>0$. (Accordingly, $z_{n,s}$ are the singular points of the above infinite integral.)

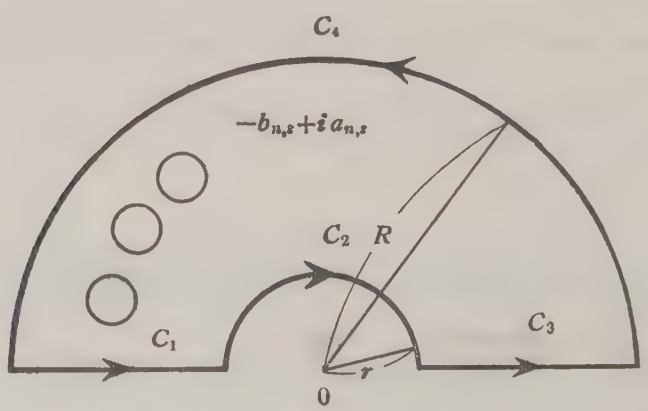

Fig. 2. The counter of integral.

Now, let us consider the following counter of integral along the curve C:

$$I=\int_c \frac{\varphi(z)}{\psi(z)}\frac{exp\{iaz\}}{z}dz, \qquad \text{where } a>0, \quad \lim_{z\to\infty}\left|\frac{\varphi(z)}{\psi(z)}\right|=1.$$

And we see $\int_c = \lim_{\substack{r\to 0\\ R\to\infty}}\left[\int_{c_1}+\int_{c_2}+\int_{c_3}+\int_{c_4}\right]$ because obviously $c=c_1+c_2+c_3+c_4$ (see Fig. 2).

From the lst and 3rd terms in the right-hand members in the above formula,

$$\lim_{\substack{r\to 0\\ R\to\infty}}\left[\int_{c_1}+\int_{c_3}\right]=\int_{-\infty}^{0}\frac{\varphi(x)}{\psi(x)}\frac{exp\{iax\}}{x}dx+\int_0^{\infty}\frac{\varphi(x)}{\psi(x)}\frac{exp\{iax\}}{x}dx$$

$$=\int_0^{\infty}\frac{\varphi(x)}{\psi(x)}\left(\frac{exp\{iax\}-exp\{-iax\}}{x}\right)dx=2i\int_0^{\infty}\frac{\varphi(x)}{\psi(x)}\frac{\sin ax}{x}dx.$$

And, from the 2nd and 4th terms,

$$\lim_{r\to 0}\left[\int_{c_2}\right]=-\pi i\lim_{z=0}\left|\frac{\varphi(z)}{\psi(z)}exp\{iaz\}\right|, \quad \int_{c_4}=i\int_0^{\pi}\frac{\varphi(z)}{\psi(z)}exp\{iaR(\cos\theta+i\sin\theta)\}d\theta.$$

But, by applying the following formura:

$$\lim_{R\to\infty}\frac{\varphi(z)}{\psi(z)}=1 \qquad \text{and} \qquad \left|exp\{iaR\cos\theta\}\right|\leqq 1,$$

we have $\int_0^{\frac{\pi}{2}} exp\{-aR\sin\theta\}d\theta<\int_0^{\frac{\pi}{2}} exp\{-aR\frac{\pi}{2}\theta\}d\theta=\frac{\pi}{2Ra}(1-exp\{-R\}).$

Consequently, we get $\lim_{R\to\infty}\left[\int_{c_4}\right]=0.$

Therefore, we obtain

$$\int_c \frac{\varphi(z)}{\psi(z)}\frac{exp\{iaz\}}{z}dz=2i\int_0^{\infty}\frac{\varphi(x)}{\psi(x)}\frac{\sin ax}{x}dx-\pi i\left[\lim_{z=0}\frac{\varphi(z)}{\psi(z)}exp\{iaz\}\right] \cdots\cdots (33)$$

On the other hand, with the aid of the Cauchy's integral theorem, we have

$$\int_c \frac{\varphi(z)}{\psi(z)}\frac{exp\{iaz\}}{z}dz=2\pi i\sum_{s=1}^{\infty}\lim_{z=-b'_{n,s}+ia'_{n,s}}\frac{z-(-b'_{n,s}+ia'_{n,s})}{\psi(z)}\frac{\varphi(z)}{z}exp\{iaz\}$$

$$=2\pi i\sum_{s=1}^{\infty}\left|\frac{\varphi(z)}{\psi'(z)}\frac{exp\{iaz\}}{z}\right|_{z=-b'_{n,s}+ia'_{n,s}}. \cdots\cdots (34)$$

Consequently, from (33) and (34) we can obtain

$$\frac{1}{\pi}\int_0^{\infty}\frac{\varphi(x)}{\psi(x)}\frac{\sin ax}{x}dx=\frac{1}{2}\lim_{z=0}\frac{\varphi(z)}{\psi(z)}exp\{iaz\}+\sum_{s=1}^{\infty}\left|\frac{\varphi(z)}{\psi'(z)}\frac{exp\{iaz\}}{z}\right|_{z=-b'_{n,s}+ia'_{n,s}}$$

(C): The calculation of $\frac{d\psi}{dz}$,

where $\psi(z)=[(p_n+iq_n)^2h^2-K]\sinh(p_n+iq_n)h+K(p_n+iq_n)h\cosh(p_n+iq_n)h$.

From the above formula $\psi(z)$, let us calculate $\frac{d\psi}{dz}$.

Namely, $\frac{d\psi}{dz}=h\{(p_n+iq_n)h\}[(p_n+iq_n)h\cosh(p_n+iq_n)h$

$$+(K+2)\sinh(p_n+iq_n)h]\times\frac{d}{dz}(p_n+iq_n).$$

And yet,

$$\frac{d}{dz}(p_n+iq_n)=\frac{d}{dz}\left\{\left[\left\{\left(\frac{\nu_x}{2\nu_z}z^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right)^2+\left(\frac{\bar{\omega}}{\nu_z}\right)^2\right\}^{\frac{1}{2}}+\left\{\frac{\nu_x}{2\nu_z}z^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right\}\right]^{\frac{1}{2}}\right.$$

$$\left.+i\left[\left\{\left(\frac{\nu_x}{2\nu_z}z^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right)^2+\left(\frac{\bar{\omega}}{\nu_z}\right)^2\right\}^{\frac{1}{2}}-\left\{\frac{\nu_x}{2\nu_z}z^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right\}\right]^{\frac{1}{2}}\right\}$$

$$=\frac{1}{2}\frac{\frac{\nu_x}{\nu_z}z}{\left\{\left(\frac{\nu_x}{2\nu_z}z^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right)^2+\left(\frac{\bar{\omega}}{\nu_z}\right)^2\right\}^{\frac{1}{2}}}\left[\left[\left\{\left(\frac{\nu_x}{2\nu_z}z^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right)^2\right.\right.\right.$$

$$\left.\left.+\left(\frac{\bar{\omega}}{\nu_z}\right)^2\right\}^{\frac{1}{2}}+\left\{\frac{\nu_x}{2\nu_z}z^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right\}\right]^{\frac{1}{2}}-i\left[\left\{\left(\frac{\nu_x}{2\nu_z}z^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right)^2\right.$$

$$\left.\left.\left.+\left(\frac{\bar{\omega}}{\nu_z}\right)^2\right\}^{\frac{1}{2}}-\left\{\frac{\nu_x}{2\nu_z}z^2+\frac{(2n+1)^2\pi^2}{2a^2}\frac{\nu_y}{\nu_z}\right\}\right]^{\frac{1}{2}}\right],$$

and moreover,

$$p_n^2+q_n^2=2\left\{\left(\frac{\nu_x}{2\nu_z}z^2+\frac{(2n+1)^2}{2a^2}\frac{\nu_y}{\nu_z}\right)^2+\left(\frac{\bar{\omega}}{\nu_z}\right)^2\right\}^{\frac{1}{2}}. \quad \cdots\cdots (36)$$

Therefore,

$$\frac{d}{dz}(p_n+iq_n)=\frac{\nu_x}{\nu_z}z\frac{p_n-iq_n}{p_n^2+q_n^2}=\frac{\frac{\nu_x}{\nu_z}\cdot z}{p_n+iq_n} \quad \cdots\cdots (37)$$

With the aid of (36) and (37), we have

$$\frac{d\psi}{dz}=h^2\frac{\nu_x}{\nu_z}z\Big[(p_n+iq_n)h\cosh(p_n+iq_n)h+(K+2)\sinh(p_n+iq_n)h\Big].$$

(D): The evaluation of $\lim_{\alpha=-b'_{n,s}+ia'_{n,s}}\dfrac{\{\alpha-(-b'_{n,s}+ia'_{n,s})\}\psi(\alpha)}{\psi(\alpha)}\dfrac{exp\{ia\alpha\}}{\alpha}$,

where $a>0$.

The formula of the above title (D) is directly able to reduce to the next formula.

Namely,

$$\lim_{\alpha=-b'_{n,s}+i\,a'_{n,s}}\frac{\alpha-(-b'_{n,s}+i\,a'_{n,s})}{\psi(\alpha)}\cdot\frac{\varphi(\alpha)}{\alpha}\cdot exp\{ia\alpha\}=\lim_{\alpha=-b'_{n,s}+i\,a'_{n,s}}\frac{\varphi(\alpha)\,exp\{ia\alpha\}}{\alpha\psi'(\alpha)}.$$

From the formula of section (C),

$$\alpha\,\psi'(\alpha)=h^2\frac{\nu_x}{\nu_z}\alpha^2\{(p_n+iq_n)h\cosh(p_n+iq_n)h+(K+2)\sinh(p_n+iq_n)h\}.$$

Therefore,

$$\lim_{\alpha=-b'_{n,s}+i\,a'_{n,s}}\alpha\,\psi'(\alpha)=h^2\frac{\nu_x}{\nu_z}(b'^2_{n,s}-a'^2_{n,s}-i\,2a'_{n,s}b'_{n,s})\{i\beta_{n,s}\cos\beta_{n,s}+(K+2)i\sin\beta_{n,s}\}$$

From (29)

$$=-ih^2\frac{\nu_x}{\nu_z}\Big[\frac{\nu_z}{\nu_x}\frac{\beta^2_{n,s}}{h^2}+\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_x}+i2\frac{\bar{\omega}}{\nu_x}\Big]\Big[\beta_{n,s}\cos\beta_{n,s}+(K+2)\sin\beta_{n,s}\Big]$$

$$=-i\beta^2_{n,s}\Big\{1+\frac{(2n+1)^2\pi^2}{a^2}\Big(\frac{\nu_y}{\nu_x}\Big)\frac{h^2}{\beta^2_{n,s}}+i2\frac{\bar{\omega}}{\nu_z}\frac{h^2}{\beta^2_{n,s}}\Big\}\Big[\beta_{n,s}+(K+2)\tan\beta_{n,s}\Big]\cos\beta_{n,s}$$

$$=-i\beta^2_{n,s}\Big\{1+\frac{(2n+1)^2\pi^2}{a^2}\frac{D_y^2h^2}{D_z^2\beta^2_{n,s}}+i\frac{2\pi^2h^2}{\beta^2_{n,s}D_z^2}\Big\}\Big[\beta_{n,s}+(K+2)\tan\beta_{n,s}\Big]\cos\beta_{n,s}$$

.....(39)

(because $\sqrt{\dfrac{\bar{\omega}}{\nu_z}}=\dfrac{\pi}{D_z}$, $\sqrt{\dfrac{\bar{\omega}}{\nu_y}}=\dfrac{\pi}{D_y}$ and $\sqrt{\dfrac{\nu_y}{\nu_z}}=\dfrac{D_y}{D_z}$)

Now, by putting

$$\varphi_1(\alpha)=(p_n+iq_n)^2h^2\sinh(p_n+iq_n)h+K(p_n+iq_n)h\{\cosh(p_n+iq_n)h-1\},$$

we can get

$$\lim_{\alpha=-b'_{n,s}+i\,a'_{n,s}} \varphi_1(\alpha)exp\{ia\alpha\}=[-\beta_{n,s}^2 i\sin\beta_{n,s}+i\,\beta_{n,s}K(\cos\beta_{n,s}-1)]exp\{-(a'_{n,s}+i\,b'_{n,s})a\}$$

$$=-i\,\beta_{n,s}[\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)]\cos\beta_{n,s}exp\{-i(a'_{n,s}+i\,b'_{n,s})a\}$$

Therefore,

$$\lim_{\alpha=-b'_{n,s}+i\,a'_{n,s}} \frac{\varphi(\alpha)}{\alpha\,\psi'(\alpha)}exp\{ia\alpha\}$$

$$=\frac{[\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)]}{1+\dfrac{(2n+1)^2\pi^2}{a^2}\dfrac{D_y^2h^2}{D_z^2\beta_{n,s}^2}+i\dfrac{2\pi^2}{\beta_{n,s}^2}\dfrac{h^2}{D_z^2}}\cdot\frac{exp\{-(a'_{n,s}+i\,b'_{n,s})a\}}{\beta_{n,s}[\beta_{n,s}+(K+2)\tan\beta_{n,s}]}\ .$$

..... (40)

Next, by putting

$$\varphi_2(\alpha)=[(p_n+iq_n)^2h^2+K\{\cosh(p_n+iq_n)h-1\}]\cosh(p_n+iq_n)(h+z)$$

$$+K[(p_n+iq_n)h-\sinh(p_n+iq_n)h]\sinh(p_n+iq_n)(h+z)$$

$$-[(p_n+iq_n)h\sinh(p_n+iq_n)h+K\{\cosh(p_n+iq_n)h-1\}],$$

we get

$$\lim_{\alpha=-b'_{n,s}+ia'_{n,s}} \varphi_2(\alpha)=-[(\beta_{n,s}^2+K)-K\cos\beta_{n,s}]\cos\beta_{n,s}\left(1+\frac{z}{h}\right)+K[i\beta_{n,s}$$

$$-i\sin\beta_{n,s}]i\sin\beta_{n,s}\left(1+\frac{z}{h}\right)+[\beta_{n,s}\sin\beta_{n,s}-K(\cos\beta_{n,s}-1)].$$

But since

$$-[(\beta_{n,s}^2+K)-K\cos\beta_{n,s}]=-\cos\beta_{n,s}[(\beta_{n,s}^2+K)\sec\beta_{n,s}-(\beta_{n,s}^2+K)+\beta_{n,s}^2]$$

$$=-\cos\beta_{n,s}[\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)]\beta_{n,s}\cot\beta_{n,s},$$

$$-[K\beta_{n,s}-K\sin\beta_{n,s}]=-\cos\beta_{n,s}[K\beta_{n,s}\sec\beta_{n,s}-K\tan\beta_{n,s}]$$

$$=-\cos\beta_{n,s}[\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)]\beta_{n,s},$$

$$\beta_{n,s}\sin\beta_{n,s}-K(\cos\beta_{n,s}-1)=\cos\beta_{n,s}[\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)],$$

we have

$$\lim_{\alpha=-b'_{n,s}+ia'_{n,s}} \varphi_2(\alpha) = -\{\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)\}\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.$$

$$\left.+\beta_{n,s}\cot\beta_{n,s}\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}\cos\beta_{n,s}. \quad \cdots\cdots (41)$$

Consequently, with the aid of the formula (41), we have the following formula:

$$\lim_{\alpha=-b'_{n,s}+ia'_{n,s}} \frac{\psi(\alpha)\,exp\{ia\alpha\}}{(p_n+iq_n)\,\alpha\,\psi'(\alpha)}$$

$$= \frac{-h\{\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)\}\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.}{\beta^3_{n,s}\left[1+\frac{(2n+1)^2\pi^2}{a^2}\frac{D_y^2}{D_z^2}\frac{h^2}{\beta^2_{n,s}}+i2\frac{\pi^2h^2}{\beta^2_{n,s}D_z^2}\right]\left[\beta_{n,s}\right.}$$

$$\left.\left.+\beta_{n,s}\cot\beta_{n,s}\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}exp\{-(a'_{n,s}+ib'_{n,s})a\}\right.$$
$$\left.+(K+2)\tan\beta_{n,s}\right]$$

$$= -\frac{1}{k}\frac{D_z}{2\pi h}\cdot\frac{2\pi^2h^2}{\beta^2{}_{n,s}D_z^2}\cdot\frac{\{\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)\}}{1+\frac{(2n+1)^2\pi^2}{a^2}\frac{D_y^2}{D_z^2}\frac{h^2}{\beta^2_{n,s}}+i2\frac{\pi^2h^2}{\beta^2_{n,s}D_z^2}}$$

$$\times\frac{\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)+\beta_{n,s}\cot\beta_{n,s}\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}exp\{-(a'_{n,s}+ib'_{n,s})a\}}{\beta_{n,s}\{\beta_{n,s}+(K+2)\tan\beta_{n,s}\}}$$

$$= -\frac{1}{k}\frac{D_z}{2\pi h}\frac{\{\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)\}}{\frac{\beta^2_{n,s}D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2}{2a^2}D_y^2+i}$$

$$\times\frac{\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)+\beta_{n,s}\cot\beta_{n,s}\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}exp\{-(a'_{n,s}+ib'_{n,s})a\}}{\beta_{n,s}\{\beta_{n,s}+(K+2)\tan\beta_{n,s}\}}$$

$$\cdots\cdots (42)$$

(E): The evaluation of $\lim_{\alpha=0}\frac{\varphi(x)}{\psi(x)}exp\{ia\alpha\}$, where $a>0$.

Let us consider the following formula:

$$\psi(\alpha)=\{(p_n+iq_n)^2h^2-K\}\sinh(p_n+iq_n)h+K(p_n+iq_n)h\cosh(p_n+iq_n)h.$$

Hence, since

$$(p_n+iq_n)^2=\frac{\nu_x}{\nu_z}\alpha^2+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i2\frac{\bar{\omega}}{\nu_z}=\frac{\nu_x}{\nu_z}\alpha^2+\frac{(2n+1)^2D_y^2}{a^2}k^2+(1+i)^2k^2,$$

we have $$\left|(p_n+iq_n)^2\right|_{\alpha=0}=k^2\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\},$$

and consequently we obtain

$$\left|p_n+iq_n\right|_{\alpha=0}=k\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}},$$

therefore

$$\left|\psi(\alpha)\right|_{\alpha=0}=\left\{\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]^2-K\right\}\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh$$

$$+K\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh,$$

.....(43)

where $$k=\sqrt{\frac{\bar{\omega}}{\nu_z}}.$$

(i) By putting

$$\varphi_1(\alpha)=(p_n+iq_n)^2h^2\sinh(p_n+iq_n)h+K(p_n+iq_n)h\{\cosh(p_n+iq_n)h-1\},$$

we get

$$\lim_{\alpha=0}\frac{\varphi_1(\alpha)}{\psi(\alpha)}\exp\{ia\alpha\}=\frac{\left[\left\{\frac{2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]^2\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}{\left\{\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]^2-K\right\}\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}\right.}$$

$$+K\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left[\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-1\right]$$

$$+(1+i)^2\Big\}^{\frac{1}{2}}kh+K\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\times\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh.$$

.....(44)

(ii) By putting

$$\varphi_2(\alpha)=\{[(p_n+iq_n)^2h^2-K]+K\cosh(p_n+iq_n)h\}\cosh(p_n+iq_n)(h+z)$$

$$+K\{(p_n+iq_n)h-\sinh(p_n+iq_n)h\}\sinh(p_n+iq_n)(h+z)$$

$$-\{(p_n+iq_n)h\sinh(p_n+iq_n)h+K[\cosh(p_n+iq_n)h-1]\},$$

we obtain

$$\lim_{\alpha=0}\varphi_2(\alpha)\,exp\{ia\alpha\}=\Big[\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}(kh)^2-K+K\cosh\Big\{\frac{(2n+1)^2D_y^2}{a^2}$$

$$+(1+i)^2\Big\}^{\frac{1}{2}}kh\times\cosh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big(1+\frac{z}{h}\Big)$$

$$+K\Big[\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh-\sinh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+1+i)^2\Big\}^{\frac{1}{2}}kh\Big]$$

$$\times\sinh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big(1+\frac{z}{h}\Big)-\Big[\Big\{\frac{(2n+1)^2D_y^2}{a^2}$$

$$+(1+i)^2\Big\}^{\frac{1}{2}}kh\sinh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh$$

$$+K\Big[\cosh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh-1\Big]\Big].$$

Then, we can get

$$\lim_{\alpha=0}\frac{\varphi_2(\alpha)exp\{ia\alpha\}}{(p_n+iq_n)\ \psi(\alpha)}=\frac{1}{\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}k}\times\Big[\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}k^2h^2-K\Big]$$

$$\times\sinh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh+K\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\times\cosh\Big\{\frac{(2n+1)^2D_y^2}{a^2}$$

$$+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big]\times\Big[\Big[\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}(kh)^2-K+K\cosh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big]$$

$$\times\cosh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big(1+\frac{z}{h}\Big)+K\Big[\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh$$

$$-\sinh\left\{\frac{(2n+1)^2}{a^2}D_y^2+(1+i)^2\right\}^{\frac{1}{2}}kh\Big]\times\sinh\left\{\frac{(2n+1)^2}{a^2}D_y^2+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)$$

$$-\Big[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\times\sinh\left\{\frac{(2n+1)^2}{a^2}D_y^2+(1+i)^2\right\}^{\frac{1}{2}}kh$$

$$+K\Big[\cosh\left\{\frac{(2n+1)^2}{a^2}D_y^2+(1+i)^2\right\}^{\frac{1}{2}}kh-1\Big]\Big]\Big]. \quad \cdots\cdots (45)$$

Thus, owing to the previous sections from (A) to (E), we have completely finished the preparations for evaluating the values of $W(x, y, z)$ and $\gamma(x, y)$.

Now, firstly let us evaluate the value of $\gamma(x, y)$.

By applying (40) and (44) to the following formula:

$$\gamma(x, y)=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}\Gamma_n(\alpha)\,d\alpha=\frac{4}{\pi}\frac{T\exp\{i\phi\}}{\rho gh}\sum_{n=0}^{\infty}\sum_{s=0}^{\infty}\frac{\sin\dfrac{(2n+1)\pi}{a}y}{2n+1}\cdot$$

$$\times\Big[\frac{1}{\pi}\int_0^{\infty}\frac{\varphi_1(\alpha)}{\psi(\alpha)}\cdot\frac{\sin\alpha x+\sin\alpha(l-x)}{\alpha}\,d\alpha\Big],$$

we have

$$\gamma(x, y)=\frac{4}{\pi}\frac{T\exp\{i\phi\}}{\rho gh}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\dfrac{(2n+1)\pi}{a}y}{2n+1}$$

$$\times\Bigg[\frac{\left[\left\{\dfrac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]^2\sinh\left\{\dfrac{(2n+1)^2D_y^2}{a^2}\right.}{\left\{\left[\left\{\dfrac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]^2-K\right\}\sinh\left\{\dfrac{(2n+1)^2D_y^2}{a^2}\right.}$$

$$\left.+(1+i)^2\right\}^{\frac{1}{2}}kh+K\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\Big[\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-1\Big]$$

$$+(1+i)^2\Big\}^{\frac{1}{2}}kh+K\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh$$

$$+\frac{1}{1+\dfrac{(2n+1)^2\pi^2}{a^2}\dfrac{D_y^2}{D_z^2}\dfrac{h^2}{\beta_{n,s}^2}+i\dfrac{2\pi^2}{\beta_{n,s}^2}\dfrac{h^2}{D_z^2}}C_{n,s}$$

$$\times\left\{\exp\{-\pi(a_{n,s}+i\,b_{n,s})\frac{x}{D_x}\}+\exp\{-\pi(a_{n,s}+i\,b_{n,s})\frac{l-x}{D_x}\}\right\}\Bigg]$$

$$\text{for}\quad 0\leqq x\leqq l, \quad \cdots\cdots (46)$$

$$\gamma(x,y)=\frac{4}{\pi}\frac{Texp\{i\phi\}}{\rho gh}\sum_{n=0}^{\infty}\sum_{s=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\left[\frac{1}{1+\frac{(2n+1)^2}{a^2}\frac{D_y^2}{D_z^2}\frac{h^2}{\beta_{n,s}^2}+i\frac{2\pi^2}{\beta_{n,s}^2}\frac{h^2}{D_z^2}}\right.$$

$$\left.\times C_{n,s}\left\{-exp\{\pi(a_{n,s}+i\,b_{n,s})\frac{x}{D_x}\}+exp\{-\pi(a_{n,s}+i\,b_{n,s})\frac{l-x}{D_x}\}\right\}\right]$$

for $x<0$, ... (47)

and

$$\gamma(x,y)=\frac{4}{\pi}\frac{Texp\{i\phi\}}{\rho gh}\sum_{n=0}^{\infty}\sum_{s=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}$$

$$\times\left[\frac{1}{1+\frac{(2n+1)^2}{a^2}\frac{D_y^2}{D_z^2}\frac{h^2}{\beta_{n,s}^2}+i\frac{2\pi^2}{\beta_{n,s}^2}\frac{h^2}{D_z^2}}C_{n,s}\right.$$

$$\left.\times\left\{exp\{-\pi(a_{n,s}+i\,b_{n,s})\frac{x}{D_x}\}-exp\{-\pi(a_{n,s}+i\,b_{n,s})\frac{x-l}{D_x}\}\right\}\right]$$

for $l<x$, ... (48)

where

$\beta_{n,s}$ are the roots of the equation $\tan\beta_{n,s}=\frac{K\beta_{n,s}}{K+\beta_{n,s}}$ which involves an infinite number of real root,

$$a_{n,s}=\left[\left\{1+\left(\frac{1}{2}\frac{D_z^2}{\pi^2}\frac{\beta_{n,s}^2}{h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)^2\right\}^{\frac{1}{2}}\right.$$

$$\left.+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)\right]^{\frac{1}{2}}$$

$$b_{n,s}=\left[\left\{1+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)^2\right\}^{\frac{1}{2}}\right.$$

$$\left.-\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)\right]^{\frac{1}{2}},$$

$$C_{n,s}=\frac{\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)}{\beta_{n,s}[\beta_{n,s}+(K+2)\tan\beta_{n,s}]},$$

$$k=\frac{\pi}{D_z}=\sqrt{\frac{\bar{\omega}}{\nu_z}},\qquad \frac{\pi}{D_x}=\sqrt{\frac{\bar{\omega}}{\nu_x}},\qquad \frac{\pi}{D_y}=\sqrt{\frac{\bar{\omega}}{\nu_y}},\qquad K=\varkappa h\ .$$

Next, let us evaluate $W(x, y, z)$. By applying the formulas (42) and (45) to the following formula:

$$W(x, y, z)=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}V_n(\alpha)\,d\alpha,$$

we have

$$W(x, y, z)=\frac{4}{\pi}\frac{Texp\{i\phi\}}{\mu k}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\left[\frac{1}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}}\right.$$

$$\times\frac{1}{\left[\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]^2-K\right]\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}$$

$$+K\left\{\frac{(2n+1)^2}{a^2}D_y^2+(1+i)^2\right\}^{\frac{1}{2}}kh\times\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh$$

$$\times\left[\left[\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]^2+K\left\{\cosh\left(\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right)^{\frac{1}{2}}kh-1\right\}\right]\right.$$

$$\times\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)+K\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right.$$

$$\left.-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]\times\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)$$

$$-\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\times\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right.$$

$$\left.\left.+K\left[\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-1\right]\right]\right]-\frac{D_z}{2\pi h}\frac{B_{n,s}}{\frac{\beta_{n,s}^2D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2}{2a^2}D_y^2+i}$$

$$\times\left[exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}+exp\{-\pi(a_{n,s}+ib_{n,s})\frac{l-x}{D_x}\}\right]\times\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.$$

$$\left.+\beta_{n,s}\cot\beta_{n,s}\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}\Bigg]$$

for $0\leqq x\leqq l$, ... (49)

$$W(x,y,z)=\frac{4}{\pi}\frac{T\exp\{i\phi\}}{\mu k}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{D_z}{2h\pi}\frac{B_{n,s}}{\frac{\beta_{n,s}^2 D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2}{2a^2}D_y^2+i}$$

$$\times\left[\exp\{\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}-\exp\{-\pi(a_{n,s}+ib_{n,s})\frac{l-x}{D_x}\}\right]\times\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.$$

$$\left.+\beta_{n,s}\cot\beta_{n,s}\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}$$

for $x<0$, ... (50)

$$W(x,y,z)=\frac{4}{\pi}\frac{T\exp\{i\phi\}}{\mu k}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{D_z}{2\pi h}\frac{B_{n,s}}{\frac{\beta_{n,s}^2 D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2}{2a^2}D_y^2+i}$$

$$\times\left[-\exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}+\exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x-l}{D_x}\}\right]\times\left[\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.$$

$$\left.+\beta_{n,s}\cot\beta_{n,s}\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right]$$

for $x>l$, ... (51)

where $B_{n,s}=\dfrac{\beta_{n,s}\tan\beta_{n,s}+K(\sec\beta_{n,s}-1)}{\beta_{n,s}\{\beta_{n,s}+(K+2)\tan\beta_{n,s}\}}$ and $K=\varkappa h$.

These series are uniformly convergent for all values of x, y, and z as far as $0\leqq y\leqq a$, and therefore can be taken as the required solutions of the problem.

From the results obtained above we see that the first part of the γ (the formula (46)) and W (the formula (49)) may be considered as the solutions (due to the consideration of $\nu_y\frac{\partial^2 W}{\partial y^2}$) corresponding to Koenuma's ones which applied Ekman's treatment to the case of a canal. (Namely, when the term of the horizontal kinematic eddy viscosity $\nu_y\frac{\partial^2 W}{\partial y^2}$ in our treatment is neglected ($\nu_y=0$, i. e. $D_y=0$), the first part is identical with the solution given by Koenuma.) It is also seen that the second part may be regarded as the correction term for the case where the wind sweeps over only a finite area of the sea surface and moreover for the case where the effect due to

the consideration of $\nu_x \frac{\partial^2 W}{\partial x^2}$ and $\nu_y \frac{\partial^2 W}{\partial y^2}$ is taken into account. Therefore, it is seen that D_x and D_y are involved in the second part. If we assume $D_y=0$, then all the parts which lie in the largest blacket in our obtained formulas (46) and (49) are identical with the solutions given by Okamoto.

Therefore, his solutions are ones which are given by permitting the exixtence of coastlines (because, he presumed the following condition: $S_y=0$) and consequently, they may be not recognized as obtained by dealing with the case of oceans which are boundless in horizontal extent.

In order to avoid this defect, we consider the case of infinite parallel straight coasts distance a apart. Then, we have need of the two boundary conditions (that the current velocity is equal to zero along the coastlines ($y=0$, $y=a$)) for W. So, we can not neglect the term of second derivative with respect to y, *i. e.* $\nu_y \frac{\partial^2 W}{\partial y^2}$, as already proposed by Prof. T. Nomitsu and Dr. T. Takegami. And the author* also formerly referred to such a view as mentioned above, when he dealt with the problem of the water elevation in an irrotational canal generated by wind.

The formulas (46) and (49) are very complicated in forms and also exceedingly inconvenient for numerical calculations and for evaluation of the qualitative characters of the obtained results.

The present author discusses with respect to these treatments in § 4 and § 5. Namely, in these paragraph he deals with the two extreme cases when $K=0$ (no bottom-friction) and $K=\infty$ (no bottom-current).

§ 4. The case of no bottom friction

In the previous paragraph, the author treated the problem on the stationary state of drift currents in a rotating canal assuming that a bottom velocity may exist and will cause a finite bottom-friction depending on the magnitude of the slip velocity.

Hence he treats here again the same problem under the assumption that there is no bottom friction at the sea bottom, so that sliping is free, as is well suggested for the case of the "mass transport" method. Now it is seen that the solutions for the case of the bottom condition: $\left|\frac{\partial W}{\partial z}\right|_{z=-h}=0$

* Saito, Y.: On the oscillation of water in a canal generated by wind action (Part 1). The Journal of the Tokyo University of Fisheries, Vol. XXXVIII, No. 1.

are given by substituting $\varkappa \to 0$ (*i. e.* $K=\varkappa h \to 0$) into the solutions (γ and W) obtained in the previous paragraph, because we can obtain $\left|\frac{\partial W}{\partial z}\right|_{z=-h}=0$ by substituting $\varkappa=0$ into the previously adopted bottom condition:

$$\left|\frac{\partial W}{\partial z}-\varkappa W\right|_{z=-h}=0.$$

By taking account of the solutions (γ and W) obtained already in our calculations, and transforming slightly the calculations on the process attaining to the previous results under the consideration of $K \to 0$, we have

$$\gamma(x, y)=\frac{T\,exp\{i\phi\}}{\rho g h} \quad \text{for} \quad 0<x<l \quad \left(\text{because} \quad 1=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\right)$$

$$=0 \quad \text{for} \quad x<0,$$

$$=0 \quad \text{for} \quad x>l. \qquad \cdots\cdots(52)$$

Next, the required current velocity becomes

$$W=\frac{4}{\pi}\frac{T\,exp\{i\phi\}}{\mu k}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}$$

$$\times\left[\frac{\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}\right.$$

$$-\frac{1}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}kh}-\frac{D_z}{2\pi h}\frac{1}{\frac{\beta_{n,s}^2D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2}{2a^2}D_y^2+i}$$

$$\left.\times\left\{exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}+exp\{-\pi(a_{n,s}+ib_{n,s})\frac{l-x}{D_x}\}\right\}\cos\beta_{n,s}\frac{z}{h}\right]$$

$$\text{for} \quad 0\leqq x\leqq l, \qquad \cdots\cdots(53)$$

$$W=\frac{4}{\pi}\frac{T\,exp\{i\phi\}}{\mu k}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{D_z}{2\pi h}\frac{1}{\frac{\beta_{n,s}^2D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2}{2a^2}D_y^2+i}$$

$$\times\left[exp\{\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}-exp\{-\pi(a_{n,s}+ib_{n,s})\frac{l-x}{D_x}\}\right]\cos\beta_{n,s}\frac{z}{h}$$

for $x<0$ ..... (54)

$$W=\frac{4}{\pi}\frac{T\,exp\{i\phi\}}{\mu k}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\dfrac{(2n+1)\pi}{a}y}{2n+1}\frac{D_z}{2\pi h}\frac{1}{\dfrac{\beta_{n,s}^2D_z^2}{2\pi^2h^2}+\dfrac{(2n+1)^2}{2a^2}D_y^2+i}$$

$$\times\left[-exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}+exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x-l}{D_x}\}\right]\cos\beta_{n,s}\frac{z}{h}$$

for $x>l$ ..... (55)

where $\tan\beta_{n,s}=0$, therefore $\sin\beta_{n,s}=0$, consequently $\beta_{n,s}=s\pi\,(s=1,2,\cdots\cdots)$,

and $$a_{n,s}=\left[\left\{1+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)^2\right\}^{\frac{1}{2}}+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)\right]^{\frac{1}{2}}$$

$$b_{n,s}=\left[\left\{1+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)^2\right\}^{\frac{1}{2}}-\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)\right]^{\frac{1}{2}}$$

$$\frac{\pi}{D_x}=\sqrt{\frac{\bar{\omega}}{\nu_x}},\qquad \frac{\pi}{D_y}=\sqrt{\frac{\bar{\omega}}{\nu_y}},\qquad k=\frac{\pi}{D_z}=\sqrt{\frac{\bar{\omega}}{\nu_z}}.$$

Hence, let us transform the formulas (53), (54), and (55) into the ones which are more simple and convenient for numerical calculations. Accordingly, let us consider the following expansion for the first term (in the largest blacket) of the right-hand members of (53).

Namely,

$$\frac{\cosh\left\{\dfrac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\dfrac{z}{h}\right)}{\sinh\left\{\dfrac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}$$

$$=\frac{1}{h}\int_{-h}^{0}\frac{\cosh\left\{\dfrac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\dfrac{z}{h}\right)}{\sinh\left\{\dfrac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\dfrac{z}{h}\right)}dz$$

$$+\frac{2}{h}\sum_{s=1}^{\infty}\left[\int_{-h}^{0}\frac{\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)}{\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}\cos\beta_{n,s}\frac{z}{h}\,dz\right]\cos\beta_{n,s}\frac{z}{h}$$

$$=\frac{1}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}+\frac{2}{h}\sum_{s=1}^{\infty}\frac{\left\{\frac{(2n+2)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}\frac{\pi}{D_z}\frac{h^2}{\beta_{n,s}}}{\left[1+\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}\frac{h^2}{\beta_{n,s}}\frac{\pi^2}{D_z^2}\right.}.$$

Therefore, the first term in the largest blacket in the formula (53) is given by

$$\frac{\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}$$

$$=\frac{1}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}kh}+2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}\frac{\frac{2\pi^2h^2}{\beta_{n,s}^2D_z^2}\cos\beta_{n,s}\frac{z}{h}}{\left[1+\left\{\frac{(2n+1)^2D^2}{a^2}+(1+i)^2\right\}\frac{h^2}{\beta_{n,s}^2}\frac{\pi^2}{D_z^2}\right]}.$$

Consequently, the first term and second term which lie in the largest blacket in the formula (53) become

$$\frac{\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}-\frac{1}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}kh}$$

$$=2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}\frac{\frac{2\pi^2h^2}{\beta_{n,s}^2D_z^2}\cos\beta_{n,s}\frac{z}{h}}{1+\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}\frac{h^2}{\beta_{n,s}^2}\frac{\pi^2}{D_z^2}}=2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}\frac{\cos\beta_{n,s}\frac{z}{h}}{\left(\frac{\beta_{n,s}^2D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2}{2a^2}D_y^2\right)+i}$$

$$=2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}\frac{exp\{-i\tan^{-1}\left(\dfrac{1}{\dfrac{\beta_{n,s}^2 D_z^2}{2\pi^2h^2}+\dfrac{(2n+1)^2}{2a^2}D_y^2}\right)\}}{\left\{1+\left(\dfrac{\beta_{n,s}^2 D_z^2}{2\pi^2h^2}+\dfrac{(2n+1)^2}{2a^2}D_y^2\right)\right\}^{\frac{1}{2}}}\cos\beta_{n,s}\frac{z}{h}\ *$$

The introduction of the new notations,

$$\theta_{n,s}=\tan^{-1}\left(\frac{1}{\dfrac{\beta_{n,s}^2 D_z^2}{2\pi^2h^2}+\dfrac{(2n+1)^2}{2a^2}D_y^2}\right)\quad\text{and}\quad R_{n,s}=\left\{1+\left(\frac{\beta_{n,s}^2 D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2}{2a^2}D_y^2\right)\right\}^{\frac{1}{2}},$$

brings (53), (54), and (55) into the following formulas.

$$W=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\dfrac{(2n+1)\pi}{a}y}{2n+1}\Big[\frac{T}{\mu k}2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}exp\{i(\phi-\theta_{n,s})\}\cos\beta_{n,s}\frac{z}{h}-\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}$$

$$\times\Big\{exp\{-\pi a_{n,s}\frac{x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\}$$

$$+exp\{-\pi a_{n,s}\frac{l-x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\}\Big\}\cos\beta_{n,s}\frac{z}{h}\Big]$$

$$\text{for}\quad 0\leqq x\leqq l,\quad\cdots\cdots(56)$$

---

* By putting

$$X=\frac{1}{\dfrac{\beta_{n,s}^2 D_z^2}{2\pi^2h^2}+\dfrac{(2n+1)^2}{2a^2}D_y^2}\quad\text{and}\quad\frac{1}{\dfrac{\beta_{n,s}^2 D_z^2}{2\pi^2h^2}+\dfrac{(2n+1)^2}{2a^2}D_y^2+i}=\frac{1}{1}\frac{1}{X+i}=I\ ,$$

we have

$$I=\frac{\dfrac{1}{X}-i}{\dfrac{1}{X^2}+1}=\frac{1}{\left(\dfrac{1}{X^2}+1\right)^{\frac{1}{2}}}\frac{\dfrac{1}{X}-i}{\left(\dfrac{1}{X^2}+1\right)^{\frac{1}{2}}}=\left(\frac{1}{X}-i\right)\frac{X}{(1+X^2)^{\frac{1}{2}}}\frac{1}{\left(\dfrac{1}{X^2}+1\right)^{\frac{1}{2}}}\ .$$

Put $\tan^{-1}X=Y$, then we have $X=\tan Y$, $\sin Y=\dfrac{X}{\sqrt{1+X^2}}$ and $\dfrac{1}{X}-i=\cot Y-i$.

$$I=\frac{1}{\left(\dfrac{1}{X^2}+1\right)^{\frac{1}{2}}}\sin Y\,(\cot Y-i)=\frac{1}{\left(\dfrac{1}{X^2}+1\right)^{\frac{1}{2}}}(\cos Y-i\sin Y)$$

Therefore,

$$I=\frac{1}{\left(\dfrac{1}{X^2}+1\right)^{\frac{1}{2}}}exp\{-i\tan^{-1}X\}=\frac{exp\{-i\tan^{-1}\left(\dfrac{1}{\dfrac{\beta_{n,s}^2 D_z^2}{2\pi^2h^2}+\dfrac{(2n+1)^2}{2a^2}D_y^2}\right)\}}{\left\{1+\left(\dfrac{\beta_{n,s}^2 D_z^2}{2\pi^2h^2}+\dfrac{(2n+1)^2}{2a^2}D_y^2\right)\right\}^{\frac{1}{2}}}.$$

$$W=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}\Big\{exp\{\pi a_{n,s}\frac{x}{D_x}+i(\phi-\theta_{n,s}+\pi b_{n,s}\frac{x}{D_x})\}$$

$$-exp\{-\pi a_{n,s}\frac{l-x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\}\Big\}\cos\beta_{n,s}\frac{z}{h}$$

for $x<0$, ... (57)

$$W=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x-l}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x-l}{D_x})\}$$

$$-exp\{-\pi a_{n,s}\frac{x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\}\Big\}\cos\beta_{n,s}\frac{z}{h}$$

for $l<x$ . ..... (58)

Separating the above formulas into the real and imaginary parts, we get

$$u=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\Big[\frac{T}{\mu k}2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}\cos(\phi-\theta_{n,s})\cos\beta_{n,s}\frac{z}{h}$$

$$-\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})$$

$$+exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\cos\beta_{n,s}\frac{z}{h}\Big]$$

$$v=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\Big[\frac{T}{\mu k}2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}\sin(\phi-\theta_{n,s})\cos\beta_{n,s}\frac{z}{h}$$

$$-\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})$$

$$+exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\Big\}\cos\beta_{n,s}\frac{z}{h}\Big]$$

for $0\leqq x\leqq l$, ... (56′)

$$u=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}\Big\{exp\{\pi a_{n,s}\frac{x}{D_x}\}\cos(\phi-\theta_{n,s}+\pi b_{n,s}\frac{x}{D_x})$$

$$-exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\Big\}\cos\beta_{n,s}\frac{z}{h}$$

$$v=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}\Big\{exp\{\pi a_{n,s}\frac{x}{D_x}\}\sin(\phi-\theta_{n,s}+\pi b_{n,s}\frac{x}{D_x})$$

$$-exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\Big\}\cos\beta_{n,s}\frac{z}{h}$$

for $x<0$, ... (57′)

$$u=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x-l}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x-l}{D_x})$$

$$-exp\{-\pi a_{n,s}\frac{x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\Big\}\cos\beta_{n,s}\frac{z}{h}$$

$$v=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}R_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x-l}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x-l}{D_x})$$

$$-exp\{-\pi a_{n,s}\frac{x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\Big\}\cos\beta_{n,s}\frac{z}{h}$$

for $x>l$ . ..... (58′)

Thus the main features of a steady current with our bottom condition can be seen at a glance from these formulas, namely:

(i) The values of current velocity change variously, oscillating with a wave number of $\frac{\pi b_{n,s}}{D_x}$ in respect of x, and take the largest value (about the absolute value) along the center line of a canal ($y=\frac{a}{2}$), as long as the values of current velocity in the place under consideration are evaluated by considering only y as a variable number. And, on the other hand, they become convergent to zero along the coastlines.

(ii) Since the convergence factors are

$$exp\{-\frac{\pi a_{n,s}}{D_x}x\} \text{ and } exp\{-\frac{\pi a_{n,s}(l-x)}{D_x}\} (\text{or } exp\{-\frac{\pi a_{n,s}(x-l)}{D_x}\}),$$

the velocity damping is very rapid for larger values of x (positive or negative value) and for larger n.

(iii) Only the our obtained results for a finite region from $x=0$ to $x=l$ involve the terms which correspond to the solution given by Ekman. But those for other regions do not involve ones. (Namely, only the term of velocity damping is involved in these formulas.)

(iv) The variations of a current velocity with the increase of z are represented by the following factor $\cos\beta_{n,s}\frac{z}{h}$ which is recognized independent of x and y.

§ 5. The case of no bottom-current

Next, the author treats the problem in this paragraph under the assumption that there can exist a no bottom current at the sea bottom, as has been often used by Ekman and others. In the present case, we see that the solutions for the case of the bottom condition: $|W|_{z=-h}=0$ are given by substituting $k\to\infty$ (*i. e.* $K\to\infty$) into the ones (γ and W) obtained previously in § 4 and § 5. Because, the infinitely intensive frictional resistance caused by the velocity at a solid surface of the sea bottom does not permit us to consider the existence of the bottom current.

Substituting $K\to\infty$ into the formulas (46), (47), and (48), we obtain the following formulas.

$$\gamma(x,y)=\frac{4}{\pi}\frac{T}{\rho gh}exp\{i\phi\}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\left[\frac{\left\{\frac{(2n+1)^2D_y^2}{a^2}\right\}^{\frac{1}{2}}}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}\right.$$

$$\left.+(1+i)^2\right\}^{\frac{1}{2}}kh\left[\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-1\right]$$

$$\times\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh$$

$$+\frac{1}{1+\frac{(2n+1)^2\pi^2}{a^2}\frac{D_y^2}{D_z^2}\frac{h^2}{\beta_{n,s}^2}+i\frac{2\pi^2}{\beta_{n,s}^2}\frac{h^2}{D_z^2}}C_{n,s}\left\{exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}\right.$$

$$+exp\{-\pi(a_{n,s}+ib_{n,s})\frac{l-x}{D_x}\}\Big\}\Big]$$

for $0 \leqq x \leqq l$, ... (59)

$$\gamma(x,y)=\frac{4}{\pi}\frac{Texp\{i\phi\}}{\rho gh}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\Big[\frac{1}{1+\frac{(2n+1)^2}{a^2}\frac{D_y^2}{D_z^2}\frac{h^2}{\beta_{n,s}^2}+i\frac{2\pi^2}{\beta_{n,s}^2}\frac{h^2}{D_z^2}}$$

$$\times C_{n,s}\Big\{-exp\{\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}+exp\{-\pi(a_{n,s}+ib_{n,s})\frac{l-x}{D_x}\}\Big\}\Big]$$

for $x<0$, ... (60)

$$\gamma(x,y)=\frac{4}{\pi}\frac{Texp\{i\phi\}}{\rho gh}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\Big[\frac{1}{1+\frac{(2n+1)^2}{a^2}\frac{D_y^2}{D_z^2}\frac{h^2}{\beta_{n,s}^2}+i\frac{2\pi^2}{\beta_{n,s}^2}\frac{h^2}{D_z^2}}$$

$$\times C_{n,s}\Big\{exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}-exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x-l}{D_x}\}\Big\}\Big]$$

for $l<x$, ... (61)

where $\beta_{n,s}$ are the roots of the equation $\tan\beta_{n,s}=\beta_{n,s}$, which involves an infinite number of real roots, and $C_{n,s}=\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}$.

Hence, let us transform these formulas into the ones which are more simple and convenient for numerical calculations. Accordingly, consider the following expansion.

$$\sinh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big(1+\frac{z}{h}\Big)=\sum_{s=1}^{\infty}A_{n,s}\Big[\int_{-h}^{0}\sinh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}$$

$$\times kh\Big(1+\frac{z}{h}\Big)\sin\beta_{n,s}\Big(1+\frac{z}{h}\Big)dz\Big]\sin\beta_{n,s}\Big(1+\frac{z}{h}\Big)$$

where $\frac{1}{A_{n,s}}=\int_{-h}^{0}\Big[\sin\beta_{n,s}\Big(1+\frac{z}{h}\Big)\Big]^2dz=\frac{h}{2}\sin^2\beta_{n,s}$,

accordingly, $A_{n,s}=\frac{2}{h}\frac{1}{\sin^2\beta_{n,s}}=\frac{2}{h}\frac{1}{\beta_{n,s}^2\cos^2\beta_{n,s}^2}$ (because $\tan\beta_{n,s}=\beta_{n,s}$).

Now, by putting $I=\int_{-h}^{0}\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)\sin\beta_{n,s}\left(1+\frac{z}{h}\right)dz$,

and carrying out this integral, we have

$$I=\frac{\beta_{n,s}h\cos\beta_{n,s}\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}kh\right.}{\beta_{n,s}^2+\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}(kh)^2}$$

$$\left.\frac{-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]}{}.$$

Therefore, the previous formula become

$$\frac{\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\times\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}$$

$$\frac{}{-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}=2\sum_{s=1}^{\infty}\frac{1}{\beta_{n,s}\cos\beta_{n,s}}\frac{1}{\beta_{n,s}^2+\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}(kh)^2}$$

$$\times\sin\beta_{n,s}\left(1+\frac{z}{h}\right),$$

accordingly

$$\frac{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}$$

$$\times\frac{\left[\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-1\right]}{-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}=\frac{\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]^2}{h}$$

$$\times\int_{-h}^{0}\frac{\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh}$$

$$\times\frac{\left(1+\frac{z}{h}\right)dz}{-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}hk}$$

$$=2\sum_{s=1}^{\infty}\frac{1}{\beta_{n,s}^2\cos\beta_{n,s}}\times\frac{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}(kh)^2}{\beta_{n,s}^2+\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}(kh)^2}\frac{\beta_{n,s}}{h}\int_{-h}^{0}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)dz.$$

By the help of $\frac{\beta_{n,s}}{h}\int_{-h}^{0}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)dz=1-\cos\beta_{n,s}$, we have

$$\text{the above formula}=2\sum_{s=1}^{\infty}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}\frac{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}(kh)^2}{\beta_{n,s}^2+\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}(kh)^2}$$

$$=2\sum_{s=1}^{\infty}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}-2\sum_{s=1}^{\infty}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}\frac{1}{1+\frac{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2}{\beta_{n,s}^2}(kh)^2}.$$

But since $kh=\frac{\pi}{D_z}h$,

the above formula

$$=2\sum_{s=1}^{\infty}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}-2\sum_{s=1}^{\infty}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}\frac{1}{1+\frac{(2n+1)^2D_y^2}{a^2}\frac{\pi^2h^2}{\beta_{n,s}^2D_z^2}+i\frac{2\pi^2h^2}{\beta_{n,s}^2D_z^2}}$$

$$=2\sum_{s=1}^{\infty}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}-2\sum_{s=1}^{\infty}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}\frac{exp\left\{-i\tan^{-1}\left(\frac{\frac{2\pi^2h^2}{\beta_{n,s}^2D_z^2}}{1+\frac{(2n+1)^2D_y^2}{a^2}\frac{\pi^2h^2}{\beta_{n,s}^2D_z^2}}\right)\right\}*}{\left\{\left(1+\frac{(2n+1)^2D_y^2}{a^2}\frac{\pi^2h^2}{\beta_{n,s}^2D_z^2}\right)^2+\left(2\frac{\pi^2h^2}{\beta_{n,s}^2D_z^2}\right)^2\right\}^{\frac{1}{2}}}.$$

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* $\frac{1}{(1+\alpha)+iX}=\frac{(1+\alpha)-iX}{(1+\alpha)^2+X^2}=\frac{1}{\left\{(1+\alpha)^2+X^2\right\}^{\frac{1}{2}}}\times\frac{1-i\left(\frac{X}{1+\alpha}\right)}{\left\{1+\left(\frac{X}{1+\alpha}\right)^2\right\}^{\frac{1}{2}}}=\frac{1}{\left\{(1+\alpha)^2+X^2\right\}^{\frac{1}{2}}}$

$$\times exp\left\{-i\tan^{-1}\left(\frac{X}{1+\alpha}\right)\right\}$$

Because, by putting $\tan\left(\frac{X}{1+\alpha}\right)=Y$, we get the following formulas:

$$\tan Y=\frac{X}{1+\alpha}\quad\text{and}\quad exp\{-iY\}=\cos Y-i\sin Y=\cos Y(1-i\sin Y)=\frac{1-i\left(\frac{X}{1+\alpha}\right)}{\left\{1+\left(\frac{X}{1+\alpha}\right)^2\right\}^{\frac{1}{2}}}$$

Here, put $\alpha=\frac{(2n+1)^2D_y^2}{a^2}\frac{\pi^2h^2}{\beta_{n,s}^2D_z^2}$ and $X=\frac{2\pi^2h^2}{\beta_{n,s}^2D_z^2}$,

then we have the above required formulas.

The introduction of the following new notations:

$$C_{n,s}=\frac{\sec\beta_{n,s}-1}{\beta_{n,s}}\ ,\quad \theta_{n,s}=\tan^{-1}\left(\frac{\dfrac{2\pi^2h^2}{\beta_{n,s}^2D_z^2}}{1+\dfrac{(2n+1)^2D_y^2}{a^2}\dfrac{\pi^2h^2}{\beta_{n,s}^2D_y^2}}\right),$$

and $$S_{n,s}=\left\{\left(1+\frac{(2n+1)^2D_y^2}{a^2}\frac{\pi^2h^2}{\beta_{n,s}^2D_z^2}\right)^2+\left(2\frac{\pi^2h^2}{\beta_{n,s}^2D_z^2}\right)^2\right\}^{-\frac{1}{2}}$$

brings (59) into the following formulas (where $a_{n,s}$ and $b_{n,s}$ are the same values as before, and $\beta_{n,s}$ are the roots of the equation $\tan\beta_{n,s}=\beta_{n,s}$ which involves an infinite number of real roots).

$$\gamma(x.y)=\frac{4}{\pi}\sum_{0=u}^{\infty}\frac{\sin\dfrac{(2n+1)\pi}{a}y}{2n+1}\left[\frac{T}{\rho hg}\left\{2\sum_{s=1}^{\infty}C_{n,s}\,exp\{i\phi\}\right\}\right.$$

$$-\frac{T}{\rho gh}\left\{2\sum_{s=1}^{\infty}C_{n,}S_{n,s}\,exp\{i(\phi-\theta_{n,s})\right\}+\frac{T}{\rho gh}\sum_{s=1}^{\infty}C_{n,s}S_{n,s}$$

$$\times\left\{exp\{-\pi a_{n,s}\frac{x}{D_x}+i(\phi-\theta_{n,s}-\pi b\frac{x}{D_x})\}\right.$$

$$\left.\left.+exp\{-\pi a_{n,s}\frac{l-x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\}\right\}\right]$$

for $0\leqq x\leqq l$, ... (62)

$$\gamma(x,y)=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\dfrac{(2n+1)\pi}{a}y}{2n+1}\left[\frac{T}{\rho gh}\sum_{s=1}^{\infty}C_{n,s}S_{n,s}\left\{-exp\{\pi a_{n,s}\frac{x}{D_x}+i(\phi-\theta_{n,s}+\pi b_{n,s}\frac{x}{D_x})\}\right.\right.$$

$$\left.\left.+exp\{-\pi a_{n,s}\frac{l-x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\}\right\}\right]$$

for $x<0$, ... (63)

$$\gamma(x,y)=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\dfrac{(2n+1)\pi}{a}y}{2n+1}\left[\frac{T}{\rho gh}\sum_{s=1}^{\infty}C_{n,}S_{n,s}\right.$$

$$\times\left\{-exp\{-\pi a_{n,s}\frac{x-l}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x-l}{D_x})\}\right.$$

$$\left.+exp\{-\pi b_{n,s}\frac{x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\}\right\}\Big]$$

for $l<x$ . ..... (64)

Separating these formulas into two parts, real and imaginary, we get

$$\gamma_x=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\Big[\frac{T}{\rho gh}\Big\{2\sum_{s=1}^{\infty}C_{n,s}\cos\phi\Big\}-\frac{T}{\rho gh}\Big\{2\sum_{s=1}^{\infty}C_{n,s}S_{n,s}\cos(\phi-\theta_{n,s})\Big\}$$

$$+\frac{T}{\rho gh}\sum_{s=1}^{\infty}C_{n,s}S_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})$$

$$+exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\Big\}\Big]$$

$$\gamma_y=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\Big[\frac{T}{\rho gh}\Big\{2\sum_{s=1}^{\infty}C_{n,s}\sin\phi\Big\}-\frac{T}{\rho gh}\Big\{2\sum_{s=1}^{\infty}C_{n,s}S_{n,s}\sin(\phi-\theta_{n,s})\Big\}$$

$$+\frac{T}{\rho gh}\sum_{s=1}^{\infty}C_{n,s}S_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})$$

$$+exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\Big\}\Big]$$

for $0\leqq x\leqq l$, ... (61′)

$$\gamma_x=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\rho gh}\Big[\sum_{s=1}^{\infty}C_{n,s}S_{n,s}\Big\{-exp\{\pi a_{n,s}\frac{x}{D_x}\}\cos(\phi-\theta_{n,s}+\pi b_{n,s}\frac{x}{D_x})$$

$$+exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\Big\}\Big]$$

$$\gamma_y=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\rho gh}\left[\sum_{s=1}^{\infty}C_{n,s}S_{n,s}\left\{-exp\{\pi a_{n,s}\frac{x}{D_x}\}\sin(\phi-\theta_{n,s}+\pi b_{n,s}\frac{x}{D_x})\right.\right.$$

$$\left.\left.+exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\right\}\right]$$

for $x<0$, ... (62′)

$$\gamma_x=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\rho gh}\left[\sum_{s=1}^{\infty}C_{n,s}S_{n,s}\left\{-exp\{-\pi a_{n,s}\frac{x-l}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x-l}{D_x})\right.\right.$$

$$\left.\left.+exp\{-\pi b_{n,s}\frac{x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\right\}\right]$$

$$\gamma_y=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\rho gh}\left[\sum_{s=1}^{\infty}C_{n,s}S_{n,s}\left\{-exp\{-\pi a_{n,s}\frac{x-l}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x-l}{D_x})\right.\right.$$

$$\left.\left.+exp\{-\pi b_{n,s}\frac{x}{D_x}\}\sin(\phi-\theta_{n,s}+\pi b_{n,s}\frac{x}{D_x})\right\}\right]$$

for $l<x$. ..... (63′)

Thus the main features of the slope of sea level with our bottom condition can be seen at a glance from the above formulas, namely:

(1) The values of surface slope change variously, oscillating with a wave number of $\frac{\pi b_{n,s}}{D_x}$ in respect of x, and take the largest value (about the absolute value) along the center line of a canal $(y=\frac{a}{2})$, as long as the values of surface slope in the place under consideration are evaluated by considering only y as a variable number. And on the other hand, they become convergent to zero along the coast-lines themselves.

(2) Since the convergence factors are

$$exp\{-\frac{\pi a_{n,s}}{D_x}x\} \text{ and } exp\{-\frac{\pi a_{n,s}(l-x)}{D_x}\} \text{ (or } exp\{-\frac{\pi a_{n,s}(x-l)}{D_x}\}),$$

the surface slope damping is very rapid for larger values of x (positive or negative value) and for larger n.

(3) Only the result obtained for a finite region from $x=0$ to $x=l$ involve the terms which correspond to the solution given by Ekman. But those for other regions do not involve ones. (Namely, only the term of surface slope damping is involved in these formulas.)

Next by introducing $K\to\infty$ into the formulas (49), (50), and (51), we can obtain the following required current velocity W.

$$W=\frac{4}{\pi}\frac{T}{\mu k}exp\{i\phi\}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\left[\frac{1}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]}\right.$$

$$\times\left[\left\{\cosh\left(\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right)^{\frac{1}{2}}kh-1\right\}\left[\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right.\right.$$

$$\left.\times\left(1+\frac{z}{h}\right)-1\right]+\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right)^{\frac{1}{2}}kh-\sinh\left(\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right)^{\frac{1}{2}}kh\right\}$$

$$\left.\times\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)\right]-\frac{D_z}{2\pi h}\frac{\beta_{n,s}}{\frac{\beta_{n,s}D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2}{2a^2}D_y^2+i}$$

$$\times\left[exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}+exp\{-\pi(a_{n,s}+ib_{n,s})\frac{l-x}{D_x}\}\right]\times\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.$$

$$\left.+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}\Bigg]$$

for $0\leqq x\leqq l$, ... (65)

$$W=\frac{4}{\pi}\frac{T}{\mu k}exp\{i\phi\}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{D_z}{2\pi h}\frac{\beta_{n,s}}{\frac{\beta_{n,s}^2D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2}{2a^2}D_y^2+i}$$

$$\times\left[exp\{\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}-exp\{-\pi(a_{n,s}+ib_{n,s})\frac{l-x}{D_x}\}\right]$$

$$\times\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}$$

for $x<0$, ... (66)

$$W=\frac{4}{\pi}\frac{T\,exp\{i\phi\}}{\mu k}\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{D_z}{2\pi h}\frac{B_{n,s}}{\frac{\beta_{n,s}D_z^2}{2\pi^2h^2}+\frac{(2n+1)^2D_y^2}{2a^2}+i}$$

$$\times\left[exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x-l}{D_x}\}-exp\{-\pi(a_{n,s}+ib_{n,s})\frac{x}{D_x}\}\right]$$

$$\times\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}$$

for $x<l$ ..... (67)

where $B_{n,s}=\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}$, and $\beta_{n,s}$ are roots of the equation $\tan\beta_{n,s}=\beta_{n,s}$ which involves an infinite number of real roots. Where, $a_{n,s}$ and $b_{n,s}$ are the same values as before.

Now, let us transform the formulas (64), (65), and (66) into the convenient forms for the numerical calculations. Regarding the first term which lie in the largest bracket in the formula (64), let us consider the following identity.

$$\frac{\pi}{D}\int_{-h}^{z}\frac{\left\{\cosh\left(\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right)^{\frac{1}{2}}kh-1\right\}\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)}{\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}\right.\right.}$$

$$+\frac{\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}\right.}{\left.\left.+(1+i)^2\right\}^{\frac{1}{2}}kh-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]}$$

$$\left.+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)dz=\frac{\left\{\cosh\left(\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right)^{\frac{1}{2}}kh-1\right\}\left[\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}\right.\right.}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}}$$

$$+\frac{\left.\left.+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)-1\right]+\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}\right.}{\times\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}\right.\right.}$$

$$+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big]\ \sinh\ \Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\ \Big(1+\frac{z}{h}\Big)$$
$$+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big]$$

.....(68)

Accordingly, let us consider the following sine series.

$$\Big\{\cosh\Big(\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big)^{\frac{1}{2}}kh-1\Big\}\sinh\ \Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big(1+\frac{z}{h}\Big)$$
$$+\Big[\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh-\sinh\ \Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big]\ \cosh\Big\{\frac{(2n+1)^2D_y^2}{a^2}$$
$$+(1+i)^2\Big\}^{\frac{1}{2}}kh\ \Big(1+\frac{z}{h}\Big)=\sum_{s=1}^{\infty}A_{n,s}\Big[\Big\{\cosh\Big(\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh-1\Big\}$$
$$\times\int_{-h}^{0}\sinh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\ \Big(1+\frac{z}{h}\Big)\ \sin\ \beta_{n,s}\frac{z}{h}dz+\Big[\Big\{\frac{(2n+1)^2D_y^2}{a^2}$$
$$+(1+i)^2\Big\}^{\frac{1}{2}}kh-\sinh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big]\int_{-h}^{0}\cosh\Big\{\frac{(2n+1)^2D_y^2}{a^2}$$
$$+(1+i)^2\Big\}^{\frac{1}{2}}kh\ \Big(1+\frac{z}{h}\Big)\sin\beta_{n,s}\frac{z}{h}dz\Big]\ \sin\beta_{n,s}\frac{z}{h},$$

where
$$\frac{1}{A_{n,s}}=\int_{-h}^{0}\Big(\sin\ \beta_{n,s}\frac{z}{h}\Big)^2dz=\frac{h}{2}\sin^2\beta_{n,s}=\frac{h}{2}\beta_{n,s}^2\cos^2\beta_{n,s},$$

(because $\tan\beta_{n,s}=\beta_{n,s}$)

therefore
$$A_{n,s}=\frac{2}{h}\frac{1}{\beta_{n,s}^2\cos^2\beta_{n,s}}.$$

And

$$\int_{-h}^{0}\sinh\ \Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big(1+\frac{z}{h}\Big)\sin\beta_{n,s}\frac{z}{h}dz$$
$$=\frac{\beta_{n,s}h\Big[\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}(kh)\cos\beta_{n,s}-\sinh\Big\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\Big\}^{\frac{1}{2}}kh\Big]}{\beta_{n,s}^2+\frac{(2n+1)^2D_y^2}{a^2}\frac{\bar{\omega}}{\nu_z}h^2+i2\frac{\bar{\omega}}{\nu_z}h^2}.$$

Consequently

$$\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)=2\sum_{s=1}^{\infty}\frac{1}{\beta_{n,s}^2\cos^2\beta_{n,s}}$$

$$\times\frac{\beta_{n,s}\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}(kh)-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]}{\beta_{n,s}^2+\frac{(2n+1)^2D_y^2}{a^2}\frac{\bar{\omega}}{\nu_z}h^2+i2\frac{\bar{\omega}}{\nu_z}h^2}\times\sin\beta_{n,s}\frac{z}{h}$$

$$=2\sum_{s=1}^{\infty}\frac{1}{\beta_{n,s}^3\cos^2\beta_{n,s}}\frac{\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}(kh)\cos\beta_{n,s}-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}\right.\right.}{1+\frac{(2n+1)^2D_y^2}{a^2}\frac{\bar{\omega}}{\nu_z}\frac{h^2}{\beta_{n,s}^2}+i2\bar{\omega}\frac{h^2}{\beta_{n,s}^2}}$$

$$\left.\left.\frac{+(1+i)^2\right\}^{\frac{1}{2}}kh\right]}{}\times\sin\beta_{n,s}\frac{z}{h}.$$

And on the other hand,

$$\int_{-h}^{0}\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)\sin\beta_{n,s}\frac{z}{h}dz$$

$$=\frac{\beta_{n,s}h\left[\cos\beta_{n,s}-\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]}{\beta_{n,s}^2+\frac{(2n+1)^2D_y^2}{a^2}\frac{\bar{\omega}}{\nu_z}h^2+i2\frac{\bar{\omega}}{\nu_z}h^2},$$

therefore

$$\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}kh\left(1+\frac{z}{h}\right)$$

$$=2\sum_{s=1}^{\infty}\frac{1}{\beta_{n,s}\cos^2\beta_{n,s}}\frac{\left[\cos\beta_{n,s}-\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]}{\beta_{n,s}^2+\frac{(2n+1)^2D_y^2}{a^2}\frac{\bar{\omega}}{\nu_z}h^2+i2\frac{\bar{\omega}}{\nu_z}h^2}\sin\beta_{n,s}\frac{z}{h}$$

$$=2\sum_{s=1}^{\infty}\frac{1}{\beta_{n,s}^3\cos^2\beta_{n,s}}\frac{\left[\cos\beta_{n,s}-\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]}{1+\frac{(2n+1)^2D_y^2}{a^2}\frac{\bar{\omega}}{\nu_z}\frac{h^2}{\beta_{n,s}^2}+i2\frac{\bar{\omega}}{\nu_z}\frac{h^2}{\beta_{n,s}^2}}\sin\beta_{n,s}\frac{z}{h},$$

accordingly

$$\left\{\cosh\left(\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right)^{\frac{1}{2}}kh-1\right\}\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)$$

$$+\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]$$

$$\times\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)$$

$$=2\sum_{s=1}^{\infty}\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right.$$

$$\left.-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]\times\frac{\cos\beta_{n,s}-1}{\beta_{n,s}^3\cos^2\beta_{n,s}}$$

$$\times\frac{1}{1+\dfrac{(2n+1)^2D_y^2}{a^2}\dfrac{\bar{\omega}}{\nu_z}\dfrac{h^2}{\beta_{n,s}^2}+i\,2\dfrac{\bar{\omega}}{\nu_z}\dfrac{h^2}{\beta_{n,s}^2}}\sin\beta_{n,s}\frac{z}{h},$$

consequently

$$\frac{1}{\left\{\dfrac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\cosh\left\{\dfrac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-\sinh\left\{\dfrac{(2n+1)^2D_y^2}{a^2}\right.}$$

$$\left.+(1+i)^2\right\}^{\frac{1}{2}}kh}\left[\left[\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-1\right]\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}\right.\right.$$

$$\left.+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)+\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}\right.\right.$$

$$\left.\left.+(1+i)^2\right\}^{\frac{1}{2}}kh\right]\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)\Bigg]$$

$$=2\sum_{s=1}^{\infty}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^3}\frac{1}{1+\dfrac{(2n+1)^2D_y^2}{a^2}\dfrac{\bar{\omega}}{\nu_z}\dfrac{h^2}{\beta_{n,s}^2}+i\,2\dfrac{\bar{\omega}}{\nu_z}\dfrac{h^2}{\beta_{n,s}^2}}\left[-\frac{\sin\beta_{n,s}\dfrac{z}{h}}{\cos\beta_{n,s}}\right].$$

Integrate the both-hand members of the above formula from $-h$ to z with respect to z, then we can obtain the following results with the aid of the formula (67).

$$\frac{1}{\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}\right.}$$

$$\left.\times kh-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]\left[\left[\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh-1\right]\right.$$

$$\times\left[\cosh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)-1\right]+\left[\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right.$$

$$\left.\left.-\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\right]\sinh\left\{\frac{(2n+1)^2D_y^2}{a^2}+(1+i)^2\right\}^{\frac{1}{2}}kh\left(1+\frac{z}{h}\right)\right]$$

$$=2\sum_{s=1}^{\infty}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^3}\frac{1}{1+\frac{(2n+1)^2D_y^2}{a^2}\frac{\bar{\omega}}{\nu_z}\frac{h^2}{\beta_{n,s}^2}+i\,2\frac{\bar{\omega}}{\nu_z}\frac{h^2}{\beta_{n,s}^2}}\frac{\pi}{D_z}\frac{h}{\beta_{n,s}}\frac{1}{\cos\beta_{n,s}}$$

$$\times\left[\cos\beta_{n,s}\frac{z}{h}-\cos\beta_{n,s}\right]$$

$$=2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}\frac{\frac{2\pi^2h^2}{D_z^2\beta_{n,s}^2}}{1+\frac{(2n+1)^2D_y^2}{a^2}\frac{\pi^2}{D_z^2}\frac{h^2}{\beta_{n,s}^2}+i\frac{2\pi^2h^2}{\beta_{n,s}^2D_z^2}}$$

$$\times\left[\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right]$$

$$=2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}\frac{1}{\left[\left(\frac{D_z^2\beta_{n,s}^2}{2\pi^2h^2}+\frac{(2n+1)^2D_y^2}{2a^2}+i\right)\right]^{\frac{1}{2}}}$$

$$\times exp\left\{-i\tan^{-1}\frac{1}{\left(\frac{D_z^2\beta_{n,s}^2}{2\pi^2h^2}+\frac{(2n+1)^2D_y^2}{2a^2}\right)}\right\}$$

$$\times\left[\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right].$$

now, by putting

$$\frac{\sec\beta_{n,s}-1}{\beta_{n,s}^2}=B_{n,s}, \quad \left[\left(\frac{D_z^2\beta_{n,s}^2}{2\pi^2h^2}+\frac{(2n+1)^2D_y^2}{2a^2}\right)^2+1\right]^{-\frac{1}{2}}=R_{n,s},$$

and $$\tan^{-1}\left(\frac{1}{\frac{D_z^2\beta_{n,s}^2}{2\pi^2h^2}+\frac{(2n+1)^2D_y^2}{2a^2}}\right)=\theta_{n,s},$$

$$=2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\,exp\{-i\theta_{n,s}\}\left[\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right].$$

Hence, by applying the underling notations as mentioned before,

$$\sqrt{\frac{\omega}{\nu_x}}=\frac{\pi}{D_x}, \sqrt{\frac{\omega}{\nu_y}}=\frac{\pi}{D_y}, \quad \sqrt{\frac{\omega}{\nu_z}}=\frac{\pi}{D_z}=k, \quad \text{and}$$

$$a_{n,s}=\left[\left\{1+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)^2\right\}^{\frac{1}{2}}+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2D_y^2}{a^2}\right)\right\}^{\frac{1}{2}},$$

$$b_{n,s}=\left[\left\{1+\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2}{a^2}D_y^2\right)^2\right\}^{\frac{1}{2}}-\left(\frac{1}{2}\frac{D_z^2\beta_{n,s}^2}{\pi^2h^2}+\frac{1}{2}\frac{(2n+1)^2D_y^2}{a^2}\right)\right\}^{\frac{1}{2}},$$

we have

$$W=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\Big[\frac{T}{\mu k}2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\,exp\{i(\phi-\theta_{n,s})\}\Big\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)$$

$$+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\Big\}-\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\}$$

$$+exp\{-\pi a_{n,s}\frac{l-x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\}\Big\}\times\Big\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)$$

$$+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\Big\}\Big]$$

for $0\leqq x\leqq l$, ... (69)

$$W=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\left\{exp\{\pi a_{n,s}\frac{x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\}\right.$$

$$\left.-exp\{-\pi a_{n,s}\frac{l-x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\}\right\}\times\left[\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.$$

$$\left.+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}\Big]$$

for $x<0$, ... (70)

$$W=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\left\{exp\{-\pi a_{n,s}\frac{x-l}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x-l}{D_x})\}\right.$$

$$\left.-exp\{-\pi a_{n,s}\frac{x}{D_x}+i(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\}\right\}\times\left[\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.$$

$$\left.+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right]$$

for $l<x$ . ..... (71)

Separating these formulas into two parts, real and imaginary, we have

$$u=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\Big[\frac{T}{\mu k}2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\cos(\phi-\theta_{n,s})\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.$$

$$\left.+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}-\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\left\{exp\{-\pi a_{n,s}\frac{x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\right.$$

$$\left.+exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\right\}\times\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.$$

$$\left.+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\right\}\Big]$$

$$v=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\Big[\frac{T}{\mu k}2\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\sin(\phi-\theta_{n,s})\left\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)\right.$$

$$+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\Big\}-\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})$$

$$+exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\Big\}\times\Big\{\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)$$

$$+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\Big\}\Big]$$

for $0\leqq x\leqq l$, ... (69′)

$$u=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\Big\{exp\{\pi a_{n,s}\frac{x}{D_x}\}\cos(\phi-\theta_{n,s}+\pi b_{n,s}\frac{x}{D_x})$$

$$-exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\Big\}\times\Big[\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)$$

$$+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\Big]$$

$$v=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\Big\{exp\{\pi a_{n,s}\frac{x}{D_x}\}\sin(\phi-\theta_{n,s}+\pi b_{n,s}\frac{x}{D_x})$$

$$-exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x})\Big\}\times\Big[\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)$$

$$+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\Big\}\Big]$$

for $x<0$, ... (70′)

$$u=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x-l}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x-l}{D_x})$$

$$-exp\{-\pi a_{n,s}\frac{x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\Big\}\times\Big[\beta_{n,s}\sin\beta_{n,s}\left(1+\frac{z}{h}\right)$$

$$+\cos\beta_{n,s}\left(1+\frac{z}{h}\right)-1\Big]$$

$$v=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{2n+1}\frac{T}{\mu k}\sum_{s=1}^{\infty}\frac{D_z}{2\pi h}B_{n,s}R_{n,s}\Big\{exp\{-\pi a_{n,s}\frac{x-l}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x-l}{D_x})$$

$$-exp\{-\pi a_{n,s}\frac{x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x})\Big\}\times\Big[\beta_{n,s}\sin\beta_{n,s}\Big(1+\frac{z}{h}\Big)$$

$$+\cos\beta_{n,s}\Big(1+\frac{z}{h}\Big)-1\Big]$$

for $x>l$ . ..... (71′)

Thus, glancing at these formulas we see the main features of a steady current with our bottom condition.

The respect specifying the case of our bottom condition (no bottom current) compared with the case of the previous one (no bottom-friction) is as follows.

Namely, the variations of a current velocity with the increase of z are represented by the following factor $\beta_{n,s}\sin\beta_{n,s}\Big(1+\frac{z}{h}\Big)+\cos\beta_{n,s}\Big(1+\frac{z}{h}\Big)-1$ which is recognized independent of x and y. The other main features are identical with those of W (in § 4) and γ (in § 5).

§ 6. Discussion and conclusion

We shall now proceed to the discussion of the physical consequences obtained by our calculations compared with Munk's solutions. It is true that our calculations must be more refined and generalized before we can draw any decisive conclusions, because they are based on a too much simplified model of the current system. But some considerable results obtained here seems to have bearing on the case to which the ordinary "mass transport" method could not give any satisfactory explanations. However, our results also of course involve the several defects. Now, let us successively explane these points which can be seen from our obtained results. We have already used the following simultaneous equations: $S_x=0$ and $S_y=0$ as the conditions of continuity. But it seems, in reality, to be correct that the following continuous equation: $\frac{\partial S_x}{\partial x}+\frac{\partial S_y}{\partial y}=0$ is applied to this case. The reason is as follows. Though the former equations may be satisfied at a certain position in our current field as used by Ekman, they may not be satisfied in general.

And the required correct condition should be expressed by the following continuous equation: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ (where w means the vertical current velocity). Therefore, integrating the above equation from $-h$ to 0 with respect to z, $\frac{\partial S_x}{\partial x}+\frac{\partial S_y}{\partial y}=0$ is given by the help of the following conditions: $|w|_{z=0}=0$ and $|w|_{z=-h}=0$. But, it seems that so long as the scale of a wind zone is not comparatively large, a picture of ocean currents (in the vicinity of a wind zone) obtained by using the former equations is not so much different from that of the latter one. Because, the interaction of the motion of sea waters of the inner part of a wind zone with the outer part of it, which is mainly played by the help of the horizontal Austausch (which may be considered as a frictional force), rapidly gives rise to the damping, in the case when any places under consideration are in some measure apart from the region of the wind zone. Accordingly, the domain in which the current of sea waters produced by wind stress appears may be comparatively confined to the small domain. So, with some approximation the same continuous equation as used by Ekman may be accepted to our case.

Strictly treating, the defect resulted from the circumstance that the former continuous equations ($S_x=0$ and $S_y=0$) are used as the conditions of continuity, is obviously exposured in the vicinity of the eastern coastline of a canal ($y=0$) (especially in the inner part of the wind zone). Namely, in this case the convergence of currents which should be drived from the surrounding region into our region under consideration does not appear. However, the divergence of currents toward the outer part of the wind zone except the above region appears only in the region of the right side of the wind zone (namely, the vicinity of the following region: $x=l$, and $0<y<a$) and on the contrary, the convergence of currents the left side (namely, the vicinity of the following region: $x=0$, $0<y<a$).

This can be easily seen by considering the physical meaning, and also can be mathematically represented by the effect of the terms

$$\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x}),\ \cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x}),\ \sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x}),$$
$$\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{l-x}{D_x}),\ \cdots\cdots\cdots\text{etc.}$$

And moreover, the current velocity rapidly decreases to zero with the increase of $|x|$ (where $x<0$ or $x>l$) by the help of the following factors

$$exp\{-\pi a_{n,s}\frac{x}{D_x}\},\ exp\{-\pi a_{n,s}\frac{x-l}{D_x}\}\ \text{(or}\ exp\{-\pi a_{n,s}\frac{l-x}{D_x}\}\text{)}.$$

Next, mainly taking up the mathematical treatment and formulation, let us discuss the solutions obtained by Dr. Munk, the use being made of the mass transport under the consiceration of the variation of the Coriolis parameter with latitude.

As stated by Munk in Journal of Meteorology (1951, Vol. 7, Nov. 2), his mathematical treatments are as follows:

Let the x axis point eastward along the equator, the y axis northward. Then the differential equation of mass transport is

$$(A\nabla^4-\beta\frac{\partial}{\partial x})\psi=-\mathrm{curl}_z\tau \quad \cdots\cdots(6)$$

under the boundary conditions

$$(\psi)=0 \quad , \quad \left(\frac{\partial\psi}{\partial x}\right)=0 \qquad \text{for} \quad x=0, \quad x=r\,; \quad \cdots\cdots(9)$$

$$(\psi)=0 \quad , \quad \left(\frac{\partial\psi}{\partial y}\right)=0 \qquad \text{for} \quad y=-s, \quad y=s\,. \quad \cdots\cdots(10)$$

Where ψ means the mass transport stream function, $\mathrm{curl}_z\tau$ the vertical component of the wind stress curl, $\beta=\frac{df}{dy}=\frac{d}{dy}(2w\sin\varphi)$, A the coefficient of horizontal mixing.

For a zonal wind circulation the stress on the sea surface in the interval $-s<y<s$ can be written as a sum of terms like

$$\tau_{xn}=a\cos ny+b\sin ny+c\,.$$

$$\text{Accordingly,} \quad \mathrm{curl}_z\tau=-\frac{\partial\tau_{xn}}{\partial y}=n(a\sin ny-b\cos ny)\,, \quad \cdots\cdots(11)$$

and the particular intergral of (6) becomes

$$\psi_{pn}=-n^{-3}A^{-1}(a\sin ny-b\cos ny)=-n^{-4}A^{-1}\mathrm{curl}_z\tau\,. \quad \cdots\cdots(12)$$

The solution of the homogeneous equation

$$(\nabla^4-k^3\frac{\partial}{\partial x})\psi_{hn}=0 \quad \cdots\cdots(13)$$

can be written as a product of $(a\sin ny-b\cos ny)$ and a function of x, where $k=3\sqrt{\beta/A}$.

$$\text{And} \quad \psi_{hn}=\psi_{pn}(r\,An^4\beta^{-1}X_n-1)\,, \quad \cdots\cdots(14)$$

where X is a function of x expressed by the underlying formula.

The complete integrals equals

$$\psi_n=\psi_{pn}+\psi_{hn}=rAn^4X_n\beta^{-1}\psi_{pn},$$

or $$\psi_n=-rX_n\beta^{-1}\mathrm{curl}_{zn}\tau,$$

where $X=-K\,exp\{-\frac{1}{2}kx\}\cos\left(\frac{\sqrt{3}}{2}kx+\frac{\sqrt{3}}{2kr}-\frac{\pi}{6}\right)+1-\frac{1}{kr}\{kx-exp\{-k(r-x)-1\}$

and $$K=\frac{2}{\sqrt{3}}-\frac{\sqrt{3}}{kr}.$$

Now, glancing at these formulas mentioned above, we can see the underlying respects compared with our solutions ($W=u+iv$) obtained in § 4. In Munk's solution, a mode of the variation of the mass transport in the eastward direction (x axis) is expressed by the formula (20) which involves the the term of

$$exp\{-\frac{1}{2}kx\}\cos\left(\frac{\sqrt{3}}{2}kx+\frac{\sqrt{3}}{2kr}-\frac{\pi}{6}\right),\quad \cdots\cdots\cdots\cdots,$$ etc., and a mode in

the northward direction (y axis) by the formula $(11)=n(a\sin ny-b\cos ny)$. In our solution which neglects the effect of the variation of the Coriolis parameter with latitude, a mode of the variation of currents in the direction of y axis (eastward direction) is expressed only by the expansion of the term of $\sin\frac{(2n+1)\pi}{a}y$ in which the damping term of exponential type is not involved, in order to satisfy the boundary conditions: $|W|_{y=0}=|W|_{y=a}$.

But, as mentioned before, a mode of the variation of currents in the direction of x axis (northward direction) is represented by the term of

$$exp\{-\pi a_{n,s}\frac{x}{D_x}\}\cos(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x}),\quad exp\{-\pi a_{n,s}\frac{x}{D_x}\}\sin(\phi-\theta_{n,s}-\pi b_{n,s}\frac{x}{D_x}),$$

…… etc., which involve the damping factor of exponential type.

Formally speaking, the solution $X(x)$ (X : a mode of the variation of mass transport in the eastward direction) is comparable to our one (a mode of the variation of currents in the northward direction). And the treantment due to the consideration of the variation of the Coriolis parameter with latitude will be in detail expressed to make the mode of the variation of currents (in the eastward direction) clear in the Appendix of Chapter 2 and the next paper.

Surmarizing from the solutions obtained by Stockman and the present author, the damping term should be not necessary given by the exponential

type, but may be given by the type of hyperbolic functions, as expressed by Stockman.

Though in Munk's solution ψ_{hn} (14) is written as a product of $a \sin ny - b$ con ny and $X(x)$, glancing at the homogeous equation (13), ψ_{hn} must involve the other functional type of y in addition to $a \sin ny - b \cos ny$. If the equation (13) can be solved to be analytically correct, the arbitary constants whose numbers are identical with those of boundary conditions (9) must be introduced into the solution obtained. But, it does not present in his solution. This respect appears to be weak point in his treatment.

Conclusion

(i) As to the thoery of stationary drift currents under the assumption that the wind sweeps all over the sea surface, it has been already discussed by several authors. And the theory of drift currents in the case where the wind blows over only a finite region of the sea surface was firstly treated by Okamoto. Neverthless in his case the sea is boundless in horizontal extent, the conditions of continuity used in the case when the sea is bounded by lands are adopted.

(ii) To avoid this defect, the present author dealt with the above problem in the case of a rotating canal bounded by two long parallel straight coasts. And taking $\nu_x \frac{\partial^2 W}{\partial x^2}$, $\nu_y \frac{\partial^2 W}{\partial y^2}$, $\nu_z \frac{\partial^2 W}{\partial z^2}$ as the terms of the horizontal and vertical kinematic eddy viscosity into account simultaneously, the equations of motion of sea waters were yielded.

(iii) The solution of this equations became very complex in comarison to ones obtained hithero by others. But, when $\nu_x = \nu_y = 0$ (namely $D_x = D_y = 0$) as a special case, we have the solutions which are identical with those of Ekman and others.

(iv) In § 4 and § 5, the theory of the cases of no bottom friction and no bottom current were treated. The main features of the results obtained are that the values of current velocity and surface slope change variously, oscillating with a wave number of $\frac{\pi b_n}{D_x}$ in respect of x. And since the convergence factors are $exp\{-\frac{\pi a_{n,s}}{D_x} x\}$ and $exp\{-\frac{\pi a_{n,s}(l-x)}{D_x}\}$ (or $exp\{-\frac{\pi a_{n,s}(x-l)}{D_x}\}$), the current velocity and surface slope damping are very rapid for larger values of x (positive or negative values) and for larger n.

(v) Lastly, we proceeded to the discussion of the physical consequence obtained by our calculations compared with Munk's solution.

Chapter 2

TRANSIENT STATE OF DRIFT CURRENTS IN THE BOUNDLESS SEA AND THE CANAL OF INFINITE DEPTH

§ 1. Introduction

The theory of drift currents in a steady state, since the Ekman's one, has been published by many authors. And as to the theory of drift currents in a transient state, it was developed by I. Fredholm (1905), J.P. van der Stock (1911), J.E. Fjeldstad (1930), Prof. K. Hidaka (1933) and Prof. T. Nomitsu (1933). But all these attempts are confirmed to the case of the drift currents discussed by taking account of the vertical mixing only as a frictional term.

Moreover, no theory of the transient state of drift currents dealt with by taking the effect of the lateral and vertical mixing into account simultaneously is yet reported by anyone, though one of the modern questions in dynamical oceanography is the role played by the lateral mixing in the movement of sea waters.

Under the consideration mentioned above, the object of the present paper is to mathematically make clear the properties of the drift currents in a transient state and to discuss in what manner the drift currents gradually develop in an ocean initially at rest and how they gain a stationay state.

For the purpose of avoiding the tediousness of mathematical treatments, however, we carry out a theoretical calculation of the above mentined problem by applying the following two special cases:

(i) A case of the drift currents in an infinitely deep ocean when the values of the coefficients of the horizontal and vertical kinematic eddy viscosity differ to each other and the ocean is boundless in horizontal extent and initially at rest, under the circumstance that the constant wind suddenly begins to blow over only a finite area of the sea surface.

(ii) A case of the drift currents in an infinitely deep canal, where the circumstances used are of the same ones as (i).

In these special cases, the terms of the horizontal pressure gradient $(g\frac{\partial\zeta}{\partial x}, \quad g\frac{\partial\zeta}{\partial y})$ resulted from by a variable surface height induced by the surface wind stress may be neglisible, as long as we deal with only the problems under the assumption that the sea is infinitely deep and the effect of a surface wind stress does not extend to the sea bottom $(=\infty)$, where ζ means the elevation of sea waters and g the accelation of gravity. The reason is as follows.

If we assume $h \to \infty$, then we are able to obtain $\frac{\partial \zeta}{\partial x} = 0$ and $\frac{\partial \zeta}{\partial y} = 0$ because the depth of oceans h has to be finite in order to become $\frac{\partial \zeta}{\partial x} \neq 0$ and $\frac{\partial \zeta}{\partial y} \neq 0$. (Namely, $\frac{\partial \zeta}{\partial x}$ and $\frac{\partial \zeta}{\partial y}$ decrease to zero as h approaches infinity.) Though the consideration mentioned above is correct in the case when the oceans are boundless in horizontal extent, it may be not always applicable to the case of an infinitely deep canal.

It seems, however, to be probable to us under the assumption that the coastlines of a canal can be roughly replaced by the starting lines of sea shelves in the oceans, when the breadth of a canal is large enough. Thus, we can plausibly obtain an approximate solution for the development of ocean currents in the vicinity of the trade and westery wind zone.

Lastly, in Appendix, short note on the theory of drift currents driven by wind in an enclosed infinitely deep ocean is published. And we treat here the above problem due to the consideration of the variation of the Coriolis parameter with latitude. But the detail of this problem is reserved for the near future.

§ 2 The case of the drift currents in an infinitely deep ocean which is boundless in horizontal extent

(i) *The equatian of motian and the initial and boundary conditions*

Let us take the x and y axes, which lie on the sea surface, as shown

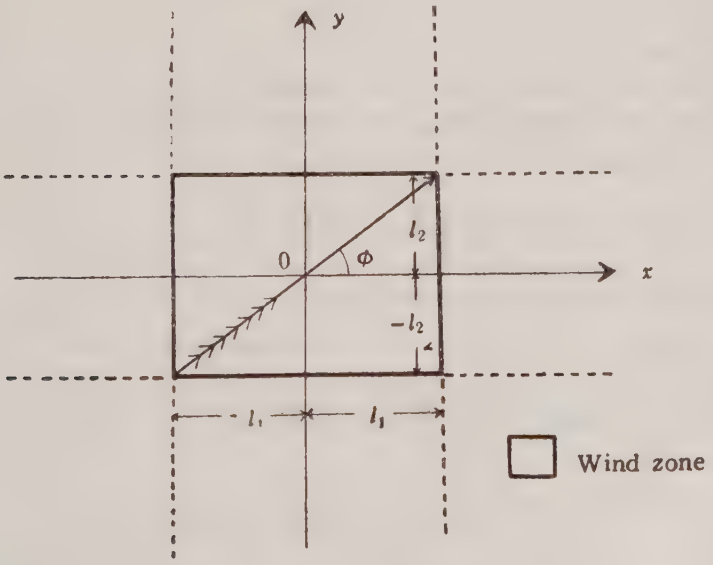

Fig. 3

by Fig. 1 and consider an infinitely deep ocean which is boundless in horizontal extent. And next, suppose that a constant (but not necessary uniform) wind sweeps over only the following finite area of a sea surface: $-l_1 \leqq x \leqq l_1$, $-l_2 \leqq y \leqq l_2$ (where $2l_1$, and $2l_2$ are the breadth of the wind region in the direction of x and y axes respectively). The z axis is taken positive upward from the sea level, the origin being placed on the center point of the wind region mentioned above. Let the following notation be used:

T: the tractive force of the wind per unit area of the sea surface, and it is uniform for $-l_1 \leqq x \leqq l_1$ and $-l_2 \leqq y \leqq l_2$, and vanishes for $x > l_1$, $x < -l_1$, $y > l_2$ and $y < -l_2$,

ϕ: the angle of T counter clockwise from the positive direction of x co-ordinate axis,

ρ: the density of sea waters,

A_x, A_y, A_z: the coefficient of the eddy viscosity of the waters in the direction of x, y, and z co-ordinate axes, respectively,

$\nu_x = \frac{A_x}{\rho}$, $\nu_y = \frac{A_y}{\rho}$, $\nu_z = \frac{A_z}{\rho}$: the coefficient of the kinematic eddy viscosity in the direction of x, y, and z co-ordinate axes, respectively,

$W = u + iv$: the complex vector of the current velocity (where u and v are the component velocity of currents in the direction of x and y co-ordinate axes, respecitvely),

ω: the angular velocity of the earth's rotation,

φ: the latitude of the place under consideration,

$\bar{\omega}$: $\omega \sin \varphi$,

t : the time.

Then the transient state of the current produced by wind in an infinitely deep ocean which was initially at rest will be represented by the solution of the equation of motion

$$\frac{\partial W}{\partial t} = \nu_x \frac{\partial^2 W}{\partial x^2} + \nu_y \frac{\partial^2 W}{\partial y^2} + \nu_z \frac{\partial^2 W}{\partial z^2} - i2\bar{\omega} W \quad \cdots\cdots (1)$$

under the initial and boundary conditions

$$|W|_{t=0} = 0 \qquad \text{when } t=0, \quad \cdots\cdots (2)$$

$$\left| A_z \frac{\partial W}{\partial z} \right|_{z=0} = T \exp\{i\phi\} \quad \text{at the sea surface } (z=0) \quad \text{for} \quad \begin{pmatrix} -l_1 < x < l_1 \\ -l_2 < y < l_2 \end{pmatrix}$$

$$= 0 \quad \text{at the sea surface } (z=0) \quad \text{for} \quad \begin{pmatrix} x > l_1,\ x < -l_1 \\ y > l_2,\ y < -l_2 \end{pmatrix}$$

$$\cdots\cdots (3)$$

$\left|\frac{\partial W}{\partial z}\right|_{z=-\infty}=0$ at the sea bottom ($z=-\infty$) for $-\infty<x<\infty$, $-\infty<y<\infty$, and simultaneously

$|W|_{z=-\infty}=0$ at the sea bottom ($z=-\infty$) for $-\infty<x<\infty$, $-\infty<y<\infty$.

..... (4)

Moreover, it can be satisfied that W, $\frac{\partial W}{\partial x}$, $\frac{\partial W}{\partial y}$, etc. are continuous at the ends of the wind region ($x=-l_1$, $x=l_1$, $y=-l_2$ and $y=l_2$) and they are zero at $x=\pm\infty$ and $y=\pm\infty$ . ..... (5)

Therefore, the required current velocity W will be given by the solution of the equation (1) that satisfies the initial condition (2) and the boundary conditions (3), (4), (5). It will, however, be very complicated to solve directly the general case in which the wind varies with the time, and so the author attacks first the case where the wind action T is constant. If the solution for the case of constant wind is known, he can at once solve also the case of varing wind by applying the Duhamel's theorem.

(ii) *The solution of differntial equation*

In order to solve the equation (1), we shall use the double Fourier's theorem.

Then, we can put

$$W=\frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty W(\lambda,\ \mu,\ z,\ t)\cos\alpha(x-\lambda)\cos\beta(y-\mu)\,d\lambda d\mu.$$

Now, by putting

$$V=\int_{-\infty}^\infty\int_{-\infty}^\infty\int W(\lambda,\ \mu,\ z,\ t)\cos\alpha(x-\lambda)\cos\beta(y-\mu)\,d\lambda d\mu$$

we have

$$W=\frac{1}{\pi^2}\int_0^\infty\int_0^\infty V\,d\alpha d\beta. \qquad \text{..... (6)}$$

Accordingly, we obtain

$$\frac{\partial W}{\partial t}=\frac{1}{\pi^2}\int_0^\infty\int_0^\infty\frac{\partial V}{\partial t}\,d\alpha d\beta$$

$$\frac{\partial^2 W}{\partial x^2}=\frac{1}{\pi^2}\int_0^\infty\int_0^\infty(-\alpha^2)\,V d\alpha d\beta \qquad \text{(by the help of the condition (5))}$$

$$\frac{\partial^2 W}{\partial y^2}=\frac{1}{\pi^2}\int_0^\infty\int_0^\infty(-\beta^2)\,V d\alpha d\beta \qquad \text{(by the help of the condition (5))}$$

and $$\frac{\partial^2 W}{\partial z^2}=\frac{1}{\pi}\int_0^\infty\int_0^\infty\frac{\partial^2 V}{\partial z^2}\,d\alpha d\beta$$

Substituting these formulas into the equation (1),

we have $$\frac{1}{\nu_z}\frac{\partial V}{\partial t}=\frac{\partial^2 V}{\partial z^2}-\left(\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\frac{i2\bar{\omega}}{\nu_z}\right)V.$$

Now, putting $\nu_z t=\tau$, the above equation becomes

$$\frac{\partial V}{\partial \tau}=\frac{\partial^2 V}{\partial z^2}-\left(\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+i\frac{2\bar{\omega}}{\nu_z}\right)V \quad \cdots\cdots (1')$$

Next, we may expand T in the same way entirely as that used for W, $\frac{\partial^2 W}{\partial x^2}$, etc.

Namely,

$$T=\frac{1}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty\int_{-\infty}^\infty T\cos\alpha(x-\lambda)\cos\beta(y-\mu)\,d\lambda d\mu$$

$$=\frac{1}{\pi^2}\int_0^\infty\int_0^\infty T(\alpha,\ \beta)\,d\alpha d\beta,$$

where

$$T(\alpha,\ \beta)=\int_{-\infty}^\infty\int_{-\infty}^\infty T\cos\alpha(x-\lambda)\cos\beta(y-\mu)\,d\lambda d\mu$$

$$=\int_{-l_1}^{l_1}\int_{-l_2}^{l_2} T\cos\alpha(x-\lambda)\cos\beta(y-\mu)\,d\lambda d\mu$$

$$=T\int_{-(x+l_1)}^{l_1-x}\int_{-(y+l_2)}^{l_2-y}\cos\alpha\lambda\cos\beta\mu\,d\lambda d\mu. \quad \cdots\cdots (7)$$

Consequently, corresponding to (2′), (3′), and (4′), the initial and boundary conditions are as follows.

$$|V|_{\tau=0}=0 \qquad \text{when } \tau=0, \quad \cdots\cdots (2')$$

$$\left|A_z\frac{\partial V}{\partial z}\right|_{z=0}=T(\alpha,\beta)\exp\{i\phi\} \text{ at the sea surface } (z=0) \text{ for } |x|<l_1$$

$$\text{and } |y|<l_2$$

$$=0 \qquad \text{at the sea surface } (z=0) \text{ for } |x|>l_1$$

$$\text{and } |y|>l_2, \quad \cdots\cdots (3')$$

$$\left|\frac{\partial V}{\partial z}\right|_{z=-\infty}=0 \qquad \text{at the sea bottom } (z=-\infty)$$

$$\text{for } -\infty<x<\infty \text{ and} -\infty<y<\infty,$$

and simultaneously $|V|_{z=-\infty}=0$ at the sea bottom $(z=-\infty)$

$$\text{for } -\infty<x<\infty \text{ and } -\infty<y<\infty.$$

$$\cdots\cdots (4')$$

Now, let us consider the following Fourier's expansion for V. Namely,

$$V=\frac{2}{\pi}\int_0^\infty d\gamma\int_{-\infty}^{\infty} V(\xi)\cos(\gamma z')\cos\gamma\xi\, d\xi$$

$$=\frac{2}{\pi}\int_0^\infty U\cos\gamma z'\, d\gamma \quad \cdots\cdots (8)$$

$$\text{where } V=\int_{-\infty}^{0} V\cos\gamma\xi\, d\xi \quad \cdots\cdots (9)$$

$$\text{and} \quad z=-z'.$$

And moreover, let us $\frac{\partial^2 V}{\partial z^2}$ expand as shown by the following form:

$$\frac{\partial^2 V}{\partial z^2}=\frac{2}{\pi}\int_0^\infty d\gamma\int_{-\infty}^{0}\frac{\partial^2 V}{\partial\xi^2}\cos\gamma\xi\, d\xi.$$

But since

$$\int_{-\infty}^{0}\frac{\partial^2 V}{\partial\xi^2}\cos\gamma\xi\,d\xi=\frac{T(\alpha,\beta)\,exp\{i\phi\}}{A_z}+\gamma\left\{\left|V\sin\gamma\xi\right|_{-\infty}^{0}-\gamma\int_{-\infty}^{0}V\cos\gamma\xi\,d\xi\right\}$$

$$=\frac{T(\alpha,\beta)\,exp\{i\phi\}}{A_z}-\gamma^2U$$

with the aid of the formulas (5′) and (9),

we obtain

$$\frac{\partial^2 V}{\partial z^2}=\frac{2}{\pi}\int_0^{\infty}\left(\frac{T(\alpha,\beta)\,exp\{i\phi\}}{A_z}-\gamma^2U\right)\cos\gamma z'\,d\gamma. \quad\cdots\cdots(10)$$

Moreover, we have

$$\frac{\partial V}{\partial\tau}=\frac{2}{\pi}\int_0^{\infty}\frac{\partial V}{\partial\tau}\cos\gamma z'\,d\gamma. \quad\cdots\cdots(11)$$

Therefore, by applying these formulas (9) and (10), the equation (1′) becomes

$$\frac{\partial U}{\partial\tau}=\frac{T(\alpha,\beta)\,exp\{i\phi\}}{A_z}-\left(\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}\right)U. \quad\cdots\cdots(12)$$

Solving the equation (12) under the following condition: $|U|_{\tau=0}$ (corresponding to (2′)), we can get

$$U=\frac{T(\alpha,\beta)\,exp\{i\phi\}}{A_z}\,\frac{1-exp\{-\left(\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}\right)\tau\}}{\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}}$$

$$=\frac{T\,exp\{i\phi\}}{A_z}\int_{-(x+l_1)}^{l_1-x}\int_{-(y+l_2)}^{l_2-y}\cos\alpha\lambda\,\cos\beta\mu\,\frac{1-exp\{-\left(\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}\right)\tau\}}{\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}}\,d\lambda\,d\mu$$

$$\cdots\cdots(13)$$

by the help of the formula (7).

By substituting the formula (13) into the formulas (8) and (6) successively, we can obtain the following required solution:

$$W=\frac{2}{\pi^3}\frac{Texp\{i\phi\}}{A_z}\int_{-(x+l_1)}^{l_1-x}\int_{-(y+l_2)}^{l_2-y}d\lambda d\mu\int_0^\infty\int_0^\infty\int_0^\infty\frac{1-exp\{-\left(\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}\right)\tau\}}{\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}}$$

$$\times\cos\alpha\lambda\cos\beta\mu\cos\gamma z'd\alpha d\beta d\gamma\,. \quad \cdots\cdots (14)$$

Now, when the wind does not vary with the time, the problem may be reduced to two simpler cases, one of these being the case of steady current and the other the case of declining current.

That is, we can divide W into two parts such that $W=W_1-W_2$, where W_1 is a function of x, y, and z only but not of t (a steady part) and W_2 is a function of x, y, z, and also t (a decaying part).

Accordingly,

$$W_1=\frac{2}{\pi^3}\frac{Texp\{i\phi\}}{A_z}\int_{-(x+l_1)}^{l_1-x}\int_{-(y+l_2)}^{l_2-y}d\lambda d\mu\int_0^\infty\int_0^\infty\int_0^\infty\frac{\cos\alpha\lambda\cos\beta\mu\cos\gamma z'}{\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}}d\alpha d\beta d\gamma\,,$$

$$\cdots\cdots (15)$$

$$W_2=\frac{2}{\pi^3}\frac{Texp\{i\phi\}}{A_z}\int_{-(x+l_1)}^{l_1-x}\int_{-(y+l_2)}^{l_2-y}d\lambda d\mu\int_0^\infty\int_0^\infty\int_0^\infty\frac{exp\{-\left(\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}\right)\tau\}}{\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}}$$

$$\times\cos\alpha\lambda\cos\beta\mu\cos\gamma z'd\alpha d\beta d\gamma\,. \quad \cdots\cdots (16)$$

The first part W_1 is nothing but the generalization of the steady current already given by Ekman, and the second W_2 corresponds to the current which would gradually decay and finally approach to zero if the wind suddenly stopped after the current had reached the steady value, and the vector difference of such two currents represents the required current in the transient state.

Next, let us evaluate the following integral for convenience to determine the value of W.

$$I=\int_0^\infty\int_0^\infty\int_0^\infty\frac{exp\{-\left(\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}\right)\tau\}}{\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}}\cos\alpha\lambda\cos\beta\mu\cos\gamma z'd\alpha d\beta d\gamma\,.$$

$$\cdots\cdots (17)$$

Differentiating the formula (17) with respect to t, we have

$$\frac{dI}{dt}=-\int_0^\infty\int_0^\infty\int_0^\infty exp\{-\left(\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}\right)\tau\}\cos\alpha\lambda\cos\beta\mu\cos\gamma z'\,d\alpha d\beta d\gamma$$

$$=-\frac{\pi\sqrt{\pi}}{8}\frac{\nu_z}{\sqrt{\nu_x\nu_y}}\frac{exp\{-\left\{i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{\lambda^2}{4\left(\frac{\nu_x}{\nu_z}\tau\right)}+\frac{\mu^2}{4\left(\frac{\nu_y}{\nu_z}\tau\right)}+\frac{z'^2}{4\tau}\right\}\}}{\tau\sqrt{\tau}}.$$

Therefore, we get

$$I=-\frac{\pi\sqrt{\pi}}{8}\frac{\nu_z}{\sqrt{\nu_x\nu_y}}\int_0^\tau\frac{exp\{-\left\{i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{\lambda^2}{4\left(\frac{\nu_x}{\nu_z}\tau\right)}+\frac{\mu^2}{4\left(\frac{\nu_y}{\nu_z}\tau\right)}+\frac{z'^2}{4\tau}\right\}\}}{\tau\sqrt{\tau}}d\tau+c. \quad \cdots\cdots(18)$$

where c is the arbitrary constant.

And the value of c is given by substituting $\tau=0$ into the above formula (18). Namely,

$$\underset{z=0}{I}=c=\int_0^\infty\int_0^\infty\int_0^\infty\frac{\cos\alpha\lambda\cos\beta\mu\cos\gamma z'}{\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\omega}{\nu_z}}d\alpha d\beta d\gamma. \quad \cdots\cdots(19)$$

Consequently, the substitution of (19) into (18) yields the following expression for the evaluation of I.

$$I=\int_0^\infty\int_0^\infty\int_0^\infty\frac{\cos\alpha\lambda\cos\beta\mu\cos\gamma z'}{\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\beta^2+\gamma^2+i\frac{2\bar{\omega}}{\nu_z}}d\alpha d\beta d\gamma$$

$$-\frac{\pi\sqrt{\pi}}{8}\frac{\nu_z}{\sqrt{\nu_x\nu_y}}\int_0^\tau\frac{exp\{-\left\{i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{\lambda^2}{4\left(\frac{\nu_x}{\nu_z}\tau\right)}+\frac{\mu^2}{4\left(\frac{\nu_y}{\nu_z}\tau\right)}+\frac{z'^2}{4\tau}\right\}\}}{\tau\sqrt{\tau}}d\tau. \quad \cdots\cdots(20)$$

By applying the formula (20), we can rewritten W_2 as the following form:

$$W_2=W_1-\frac{T\exp\{i\phi\}}{4\pi\sqrt{\pi A_z}}\frac{\nu_z}{\sqrt{\nu_x\nu_y}}\int_{-(x+l_1)}^{l_1-x}d\lambda\int_{-(y+l_2)}^{l_2-y}d\mu\frac{\exp\{-\left\{i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{\lambda^2}{4\left(\frac{\nu_x}{\nu_z}\tau\right)}+\frac{\mu^2}{4\left(\frac{\nu_y}{\nu_z}\tau\right)}+\frac{z'^2}{4\tau}\right\}\}}{\tau\sqrt{\tau}}d\tau.$$

Therefore, the required current velocity $W=W_1-W_2$ is as follows:

$$W=\frac{T}{4\pi}\frac{\exp\{i\phi\}}{\sqrt{\pi}A_z}\frac{\nu_z}{\sqrt{\nu_x\nu_y}}\int_0^\tau\frac{\exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{z'^2}{4\tau}\right)\}}{\tau\sqrt{\tau}}d\tau\int_{-(x+l_1)}^{l_1-x}\exp\{-\frac{\lambda^2}{4\left(\frac{\nu_x}{\nu_z}\tau\right)}\}d\lambda$$

$$\times\int_{-(y+l_2)}^{l_2-y}\exp\{-\frac{\mu^2}{4\left(\frac{\nu_y}{\nu_z}\tau\right)}\}d\mu$$

But since

$$\int_{-(x+l_1)}^{l_1-x}\exp\{-\frac{\lambda^2}{4\frac{\nu_x}{\nu_z}\tau}\}d\lambda=\sqrt{\pi}\sqrt{\frac{\nu_x}{\nu_z}\tau}\left[\Phi\left(\frac{l_1-x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)+\Phi\left(\frac{l_1+x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]$$

$$\int_{-(y+i_2)}^{l_2-y}\exp\{-\frac{\mu^2}{4\frac{\nu_y}{\nu_z}\tau}\}d\mu=\sqrt{\pi}\sqrt{\frac{\nu_y}{\nu_z}\tau}\left[\Phi\left(\frac{l_2-y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)+\Phi\left(\frac{l_2+y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

where $\Phi(x)=\frac{2}{\sqrt{\pi}}\int_0^x\exp\{-x^2\}dx$ (Error function),

we have

$$W=\frac{T\exp\{i\phi\}}{4\sqrt{\pi}A_z}\int_0^\tau\left[\Phi\left(\frac{l_1-x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)+\Phi\left(\frac{l_1+x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]\left[\Phi\left(\frac{l_2-y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)+\Phi\left(\frac{l_2+y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

$$\times\frac{\exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{z'^2}{4\tau}\right)\}}{\sqrt{\tau}}d\tau. \qquad \cdots\cdots(21)$$

In the formula (21), if l_1 and l_2 take the value of infinity and ϕ takes that of $-\frac{\pi}{2}$, the formula will be reduced to the following expression:

$$W=\frac{iT}{\sqrt{\rho A_z \pi}}\int_0^{\tau}\frac{exp\{-\left(i2\bar{\omega}\tau+\frac{z'^2}{4\nu_z\tau}\right)\}}{\sqrt{\tau}}d\tau. \qquad \cdots\cdots (22)$$

It is seen that this above formula (22) is identical with the Fredholm's solution for an infinitely deep ocean. (Accordingly, (21) may be recognized as the generalization of the solution given by Fredholm.) And it also shows the fact that the horizontal mixing disappears when the wind blows all over the sea surface. And, these appear the solutions corresponding to the following nine cases since the wind region lies on the following domain: $|x| \leqq l_1$, $|y| \leqq l_2$.

Let us specify the underlying cases of the wind region by the numbers from (I) to (IX). (See Fig. 4.)

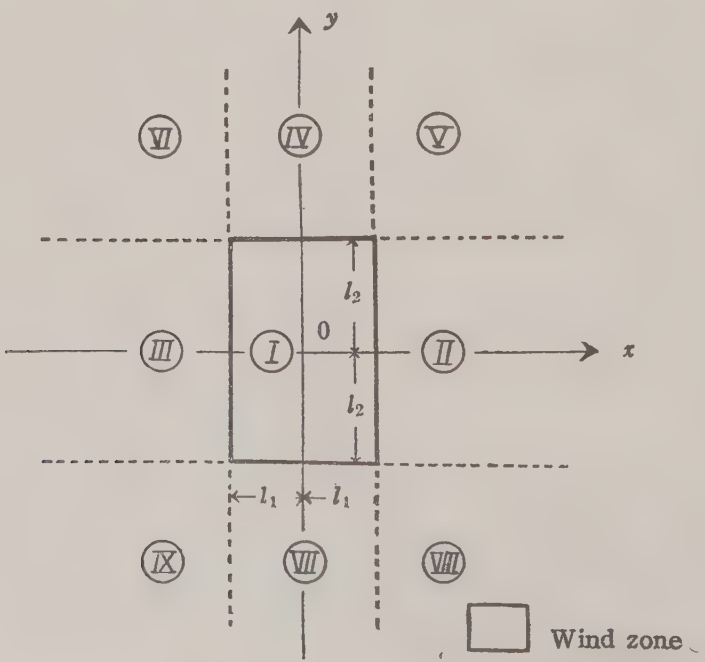

Fig. 4

(I) $|x| \leqq l_1 \quad |y| \leqq l_2$

(II) $x > l_1 \quad |y| \leqq l_2$

(III) $x < -l_1 \quad |y| \leqq l_2$

(IV) $|x| \leqq l_1 \quad y > l_2$

(V) $x > l_1 \quad y > l_2$

(IV) $x < -l_1 \quad y > l_2$

(VII) $|x| \leqq l_1 \quad y < -l_2$

(VIII) $x > l_1 \quad y < -l_2$

(IX) $x < -l_1 \quad y < -l_2$

(I)

$$W = \frac{T e x p\{i\phi\}}{4\sqrt{\pi}\,A_z} \int_0^\tau \left[\Phi\left(\frac{l_1 - x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right) + \Phi\left(\frac{l_1 + x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]\left[\Phi\left(\frac{l_2 - y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right) + \Phi\left(\frac{l_2 + y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

$$\times \frac{exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau + \frac{z'^2}{4\tau}\right)\}}{\sqrt{\tau}}\, d\tau$$

(II)

$$W = \frac{T e x p\{i\phi\}}{4\sqrt{\pi}\,A_z} \int_0^\tau \left[\Phi\left(\frac{l_1 + x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right) - \Phi\left(\frac{x - l_1}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]\left[\Phi\left(\frac{l_2 - y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right) + \Phi\left(\frac{l_2 + y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

$$\times \frac{exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau + \frac{z'^2}{4\tau}\right)\}}{\sqrt{\tau}}\, d\tau$$

(III)

$$W = \frac{T e x p\{i\phi\}}{4\sqrt{\pi}\,A_z} \int_0^\tau \left[\Phi\left(\frac{l_1 - x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right) - \Phi\left(\frac{l_1 + x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]\left[\Phi\left(\frac{l_2 - y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right) + \Phi\left(\frac{l_2 + y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

$$\times \frac{exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau + \frac{z'^2}{4\tau}\right)\}}{\sqrt{\tau}}\, d\tau$$

(IV)

$$W = \frac{T e x p\{i\phi\}}{4\sqrt{\pi}\,A_z} \int_0^\tau \left[\Phi\left(\frac{l_1 - x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right) + \Phi\left(\frac{l_1 + x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]\left[\Phi\left(\frac{l_2 + y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right) - \Phi\left(\frac{y - l_2}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

$$\times \frac{exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau + \frac{z'^2}{4\tau}\right)\}}{\sqrt{\tau}}\, d\tau$$

(V)

$$W=\frac{T\exp\{i\phi\}}{4\sqrt{\pi}\,A_z}\int_0^\tau\left[\Phi\left(\frac{l_1+x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)-\Phi\left(\frac{x-l_1}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]\left[\Phi\left(\frac{l_2+y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)-\Phi\left(\frac{y-l_2}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

$$\times\frac{\exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{z'^2}{4\tau}\right)\}}{\sqrt{\tau}}\,d\tau$$

(VI)

$$W=\frac{T\exp\{i\phi\}}{4\sqrt{\pi}\,A_z}\int_0^\tau\left[\Phi\left(\frac{l_1-x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)-\Phi\left(\frac{l_1+x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]\left[\Phi\left(\frac{l_2+y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)-\Phi\left(\frac{y-l_2}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

$$\times\frac{\exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{z'^2}{4\tau}\right)\}}{\sqrt{\tau}}\,d\tau$$

(VII)

$$W=\frac{T\exp\{i\phi\}}{4\sqrt{\pi}\,A_z}\int_0^\tau\left[\Phi\left(\frac{l_1-x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)+\Phi\left(\frac{l_1+x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]\left[\Phi\left(\frac{l_2-y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)-\Phi\left(\frac{l_2+y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

$$\times\frac{\exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{z'^2}{4\tau}\right)\}}{\sqrt{\tau}}\,d\tau$$

(VIII)

$$W=\frac{T\exp\{i\phi\}}{4\sqrt{\pi}\,A_z}\int_0^\tau\left[\Phi\left(\frac{l_1+x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)-\Phi\left(\frac{x-l_1}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]\left[\Phi\left(\frac{l_2-y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)-\Phi\left(\frac{l_2+y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

$$\times\frac{\exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{z'^2}{4\tau}\right)\}}{\sqrt{\tau}}\,d\tau$$

(IX)

$$W=\frac{T\exp\{i\phi\}}{4\sqrt{\pi}\,A_z}\int_0^\tau\left[\Phi\left(\frac{l_1-x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)-\Phi\left(\frac{l_1+x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]\left[\Phi\left(\frac{l_2-y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)-\Phi\left(\frac{l_2+y}{2\sqrt{\frac{\nu_y}{\nu_z}\tau}}\right)\right]$$

$$\times\frac{\exp\{-\left(i\frac{2\bar{\omega}}{\nu_z}\tau+\frac{z'^2}{4\tau}\right)\}}{\sqrt{\tau}}\,d\tau$$

Knowing the current which should be produced by a constant wind, we can at once derive the formula for the current when the wind varies with the time (T is a function of t such that $T=T(t)$), by applying the Duhamel's theorem.

Namely, when $T=T(t)$ the required solution $\overline{W}$ becomes

$$\overline{W}=\int_0^t \frac{\partial W(x,y,z,t-\theta,T(\theta)}{\partial t} d\theta,$$

where W means the solution for the case when T=constant.
The numerical calculations of these formulas will be reserved for the near future.

§ 3. The case of an infinitely deep canal

In this case, first consider a rotating canal of infinite depth bounded by two long parallel straight coasts distance a apart and suppose that a constant wind begins to blow suddenly over a finite region of the canal surface from $x=0$ to $x=l$ (where the canal waters are initially at rest).

Take the x-axis along the coasts on the sea surface, the y-axis perpendiculary to the coasts and the z-axis vertically downwards from a level surface (assumed plane), the other notations being the same as those in our previous paragraph of this chapter. (See Fig. 5.)

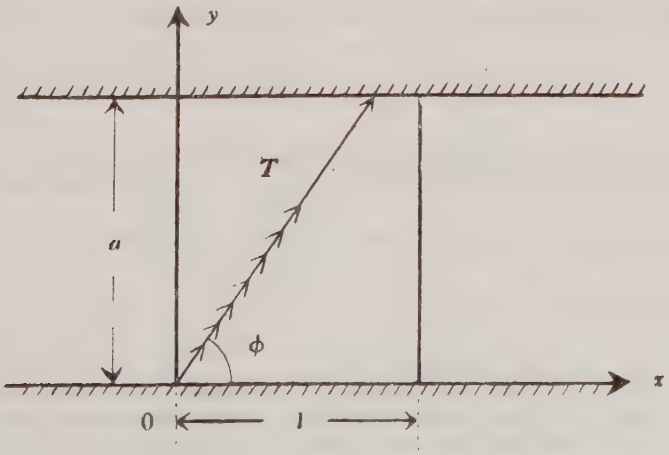

Fig. 5

(i): *The foundamental equation and the initial and boundary condition*

The dynamical equation of the transient state of drift currents in our case, taking the horizontal and vertical Austausch into account simultaneously and neglecting the nonlinear terms and the horizontal pressure gradients*, are

$$\frac{\partial W}{\partial t}=\nu_x\frac{\partial^2 W}{\partial x^2}+\nu_y\frac{\partial^2 W}{\partial y^2}+\nu_z\frac{\partial^2 W}{\partial z^2}+i2\bar{\omega}W. \quad \cdots\cdots(1)$$

The initial condition is $|W|_{t=0}=0$, when $t=0$. $\cdots\cdots(2)$

The boundary conditions to be satisfied on the surface ($z=0$) and at the bottom ($z=-\infty$) are

$$\left|A_z\frac{\partial W}{\partial z}\right|_{z=0}=Texp\{i\phi\} \quad \text{for } 0\leqq x\leqq l$$
$$=0 \quad \text{for } x<0,\ x>l \qquad \cdots\cdots(3)$$

$$\left|\frac{\partial W}{\partial z}\right|_{z=-\infty}=0 \quad \text{for } -\infty<x<\infty$$

and simultaneously $|W|_{z=-\infty}=0$ for $-\infty<x<\infty$. $\cdots\cdots(4)$

Here the coefficients of kinematic eddy viscosity (ν_x, ν_y, ν_z) are supposed to be constant. The conditions to be satisfied along the coasts are also necessary. These are simply that there is no water flow across and along these coasts. If the coasts consist of vertical cliffs, we have,

$$|W|_{y=0}=|W|_{y=a}=0 \quad \cdots\cdots(5)$$

Moreover, it must be satisfied that W, $\frac{\partial W}{\partial x}$, etc. are continuous at the ends of the wind region ($x=0$, $x=l$) $\cdots\cdots(6)$

and they are zero at $x=\pm\infty$. $\cdots\cdots(7)$

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* As stated in the previous introduction of this chapter, the horizontal pressure gradients caused by a variable surface height induced by the surface wind stress may be considered neglisible, under the assumption that the coastlines of a canal can be roughly replaced by the starting lines of sea shelves in the ocean, by the reason of treating the case of the infinity of the depth.

And, let us assume that the distribution of wind stress is represent by the following form $T=T_0 y(a-y)$, where T_0 means the value of wind stress at the center of a wind region.

(ii): *The solution of differential equation.*

In order to solve the equation (1), we shall use Stokes method. Let W be expressed by Fourier's integral for x, and next with Fourier's sine series for y under the consideration of the condition (5).
Then, we have

$$W=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}V_n(\alpha)\,d\alpha, \qquad \cdots\cdots(8)$$

where

$$V_n=\int_{-\infty}^{\infty}\int_0^a W(\lambda,\ \mu,\ z,\ t)\cos\alpha(x-\lambda)\sin\frac{(2n+1)\pi}{a}\mu\,d\lambda\,d\mu\,.$$

Moreover, expressing $\frac{\partial^2 W}{\partial x^2}$, $\frac{\partial^2 W}{\partial y^2}$, $\frac{\partial^2 W}{\partial z^2}$, $\frac{\partial W}{\partial t}$ and T similary as above,

$$\frac{\partial^2 W}{\partial x^2}=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}(-\alpha^2)\,V_n(\alpha)\,d\alpha$$

by the help of the conditions (6) and (7),

$$\frac{\partial^2 W}{\partial y^2}=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}\left\{-\frac{(2n+1)^2\pi^2}{a^2}\right\}V_n(\alpha)\,d\alpha$$

with the aid of the condition (5),

$$\frac{\partial^2 W}{\partial z^2}=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}\frac{\partial^2 V_n}{\partial z^2}\,d\alpha,$$

$$\frac{\partial W}{\partial t}=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}\frac{\partial V_n}{\partial t}\,d\alpha,$$

and also

$$T=\frac{2}{\pi a}\sum_{n=0}^{\infty}\sin\frac{(2n+1)\pi}{a}y\int_0^{\infty}T_n(\alpha)\,d\alpha,$$

where $T_n(\alpha)=\int_{-\infty}^{\infty}\int_0^a T_0\mu(a-\mu)\sin\frac{(2n+1)\pi}{a}\mu\cos\alpha(x-\lambda)\,d\lambda d\mu$

$$=\int_0^a T_0\mu(a-\mu)\sin\frac{(2n+1)\pi}{a}\mu d\mu\int_0^l\cos\alpha(x-\lambda)\,d\lambda$$

$$=\frac{4T_0a^3}{\pi^3}\frac{1}{(2n+1)^3}\int_{-x}^{l-x}\cos\alpha\lambda d\lambda. \qquad \cdots\cdots(8)$$

Therefore, substitute these above formulas into the equation (1), and equate the coefficients of the n-th term in the both sides for $\sin\frac{(2n+1)\pi}{a}y$, then we have

$$\frac{\partial V_n}{\partial\tau}=\frac{\partial^2V_n}{\partial z^2}-\left\{\frac{\nu_x}{\nu_z}\alpha^2+\frac{\nu_y}{\nu_z}\frac{(2n+1)^2\pi^2}{a^2}+i\frac{2\bar{\omega}}{\nu_z}\right\}V_n, \qquad \cdots\cdots(1')$$

where $\nu_z t=\tau$.

Corresponding to (2'), (3'), and (4'), the initial and boundary conditions are as follows:

$$|V_n|_{t=0}=0 \qquad \text{when} \qquad t=0, \qquad \cdots\cdots(2')$$

$$\left|A_z\frac{\partial V_n}{\partial z}\right|_{z=0}=T_n(\alpha)\,exp\{i\phi\} \qquad \text{for } 0\leqq x\leqq l$$
$$=0 \qquad \text{for } x>l,\ x<0 \qquad \Big\}\cdots\cdots(3')$$

$$\left|\frac{\partial V_n}{\partial z}\right|_{z=\infty}=0 \qquad \text{for } -\infty<x<\infty$$
and simultaneously $|V|_{z=\infty}=0$ for $-\infty<x<\infty$ $\Big\}\cdots\cdots(4')$.

Again, let us consider the following Fourier's expansion for V.

Namely,

$$V_n(\tau,z)=\frac{2}{\pi}\int_0^\infty U(\beta,\tau)\cos\beta z'd\beta \qquad \cdots\cdots(9)$$

where $U=\int_{-\infty}^0 V_n(\xi)\cos\beta\xi d\xi$ $\cdots\cdots(10)$

and $z=-z'$.

Similarly, $\frac{\partial^2 V_n}{\partial z^2}=\frac{2}{\pi}\int_0^\infty d\beta\int_{-\infty}^0\frac{\partial^2 V_n}{\partial z^2}\cos\beta\xi\cos\beta z'd\xi$

$$=\frac{2}{\pi}\int_0^\infty\left\{\frac{T_n(\alpha)}{A_z}exp\{i\phi\}-\beta^2U(\beta)\right\}\cos\beta z'd\beta$$

with the aid of (3′), (4′), and (7).

And $\frac{\partial V_n}{\partial\tau}=\frac{2}{\pi}\int_0^\infty d\beta\int_{-\infty}^0\frac{\partial U}{\partial\tau}\cos\beta z'd\beta.$

Substituting these above two formulas into the equation (1′),

we have

$$\frac{\partial U}{\partial\tau}=\frac{T_n(\alpha)}{A_z}exp\{i\phi\}-\left(\frac{\nu_x}{\nu_z}\alpha^2+\beta^2+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right)U, \quad \cdots\cdots (11)$$

under the following condition: $|U|_{\tau=0}=0.$ $\cdots\cdots (12)$

Then, the solution of the equation (9) under the condition (10) is as follows:

$$U=\frac{T_n(\alpha)}{A_z}exp\{i\phi\}\frac{1-exp\left\{-\left\{\frac{\nu_x}{\nu_z}\alpha^2+\beta^2+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}\tau\right\}}{\frac{\nu_x}{\nu_z}\alpha^2+\beta^2+\left\{\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}}$$

$\cdots\cdots (13)$

Substitute (13) into (9) and (8), then we have the following required current velocity W.

Namely,

$$W=\frac{8T_0a^2exp\{i\phi\}}{A_z\pi^4}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{(2n+1)^3}$$

$$\times\frac{2}{\pi}\int_{-x}^{l-x}d\lambda\int_0^\infty\int_0^\infty\frac{1-exp\left\{-\left\{\frac{\nu_x}{\nu_z}\alpha^2+\beta^2+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}\tau\right\}}{\frac{\nu_x}{\nu_z}\alpha^2+\beta^2+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}}$$

$$\times\cos\alpha\lambda\cos\beta z'd\alpha d\beta. \quad \cdots\cdots (14)$$

Now, divide the above W (14) into two parts such that

$$W=W_1-W_2,$$

where W_1 is a function of x, y, and z but not of t (namely, W_1 is a solution for stationary state), and W_2 is a function of x, y, z, and t (namely, W_2 is a solution for decaying state; that is, it is a state corresponding to the case when the wind stress abruptly stopped).

Consequently, Steady part W_1: Now the part W_1 is nothing but the generalization of the solution given by Ekman, i. e.

$$W_1=\frac{8T_0a^2\,exp\{i\phi\}}{A_z\pi}\sum_{n=0}^{\infty}\frac{\sin\dfrac{(2n+1)\pi}{a}y}{(2n+1)^3}\,\frac{2}{\pi}\int_{-\infty}^{l-\infty}d\lambda\int_0^{\infty}\int_0^{\infty}\frac{\cos\alpha\lambda\cos\beta z'\,d\alpha d\beta}{\dfrac{\nu_x}{\nu_z}\alpha^2+\beta^2+\dfrac{(2n+1)^2\pi^2}{a^2}\dfrac{\nu_y}{\nu_z}+i\dfrac{2\bar{\omega}}{\nu_z}}. \qquad \cdots\cdots\cdots\cdots (15)$$

Decaying part W_2: The second part W_2 corresponds to a case where the above steady current will decay if the wind suddenly disappears, and it differs from the required transient part W (only in its initial value,) i. e.

$$W_2=\frac{8T_0a^2exp\{i\phi\}}{A_z\pi^4}\sum_{n=0}^{\infty}\frac{\sin\dfrac{(2n+1)\pi}{a}y}{(2n+1)^3}\,\frac{2}{\pi}\int_{-\infty}^{l-\infty}d\lambda\int_0^{\infty}\int_0^{\infty}\frac{exp\left\{-\left\{\dfrac{\nu_x}{\nu_z}\alpha^2+\beta^2+\dfrac{(2n+1)^2\pi^2}{a^2}\dfrac{\nu_y}{\nu_z}+i2\dfrac{\bar{\omega}}{\nu_z}\right\}\tau\right\}}{\dfrac{\nu_x}{\nu_z}\alpha^2+\beta^2+\dfrac{(2n+1)^2\pi^2}{a^2}\dfrac{\nu_y}{\nu_z}+i2\dfrac{\bar{\omega}}{\nu_z}}$$

$$\times\cos\alpha\lambda\cos\beta z'\,d\alpha\,d\beta. \qquad \cdots\cdots\cdots\cdots (16)$$

Hence now, let us prepare for evaluating the triple integral in the formulas (15) and (16).

(A): First evaluate the underlying double integral.

Namely,

$$I_1=\frac{2}{\pi}\int_0^{\infty}\int_0^{\infty}\frac{\cos\alpha\lambda\cos\beta z'\,d\alpha\,d\beta}{\dfrac{\nu_x}{\nu_z}\alpha^2+\left[\beta^2+\left\{\dfrac{(2n+1)^2}{a^2}\dfrac{\nu_y}{\nu_z}+i\dfrac{2\bar{\omega}}{\nu_z}\right\}\right]}$$

$$=\frac{1}{\sqrt{\dfrac{\nu_x}{\nu_z}}}\int_0^{\infty}\frac{exp\left\{-\lambda\sqrt{\dfrac{\nu_z}{\nu_x}\left[\beta^2+\left\{\dfrac{(2n+1)^2}{a^2}\dfrac{\nu_y}{\nu_z}+i\dfrac{2\bar{\omega}}{\nu_z}\right\}\right]^{\frac{1}{2}}}\right\}}{\left[\beta^2+\left\{\dfrac{(2n+1)^2}{a^2}\dfrac{\nu_y}{\nu_z}+i\dfrac{2\bar{\omega}}{\nu_z}\right\}\right]^{\frac{1}{2}}} \qquad \cdots\cdots\cdots\cdots (17)$$

(with the aid of the following equallity: $\int_0^\infty \frac{\cos\alpha\lambda}{\alpha^2+u^2}d\alpha=\frac{\pi}{2}\cdot\frac{exp\{-\lambda u\}}{u}$).

And moreover, let us consider the new following equation to evaluate (17):

$$f(\lambda,z,k)=\int_0^\infty \frac{exp\{-\lambda\sqrt{\beta^2+k^2}\cos\beta z\}}{\sqrt{\beta^2+k^2}}d\beta. \qquad \cdots\cdots (18)$$

The introduction of the new variable number, $\lambda^2+z^2=t^2$, brings (18) into the following equation.

Namely,

$$f(\lambda,z,k)=\varphi(\lambda,t,k)=\int_0^\infty \frac{exp\{-\lambda\sqrt{\beta^2+k^2}\cos(\beta\sqrt{t^2-\lambda^2})\}}{\sqrt{\beta^2+k^2}}d\beta.$$

Differentiating the above $\varphi(\lambda,t,k)$ with respect to λ, we get

$$\frac{\partial\varphi}{\partial\lambda}=-\int_0^\infty exp\{-\lambda\sqrt{\beta^2+k^2}\}\cos\beta(\sqrt{t^2-\lambda^2})\,d\beta$$

$$+\int_0^\infty \frac{exp\{-\lambda\sqrt{\beta^2+k^2}\}}{\sqrt{\beta^2+k^2}}\,\frac{\lambda\beta\sin(\beta\sqrt{t^2-\lambda^2})}{\sqrt{t^2-\lambda^2}}\,d\beta.$$

But since

the second term of $\frac{\partial\varphi}{\partial\lambda}=-\int_0^\infty \frac{d}{d\beta}\,exp\{-\lambda\sqrt{\beta^2+k^2}\}\,\frac{\sin(\beta\sqrt{t^2-\lambda^2})}{\sqrt{t^2-\lambda^2}}d\beta$

$$=-\left[exp\{-\lambda\sqrt{\beta^2+k^2}\}\,\frac{\sin(\beta\sqrt{t^2-\lambda^2})}{\sqrt{t^2-\lambda^2}}\right]_{\beta=0}^{\beta=\infty}$$

$$+\int_0^\infty exp\{-\lambda\sqrt{\beta^2+k^2}\}\cos(\beta\sqrt{t^2-\lambda^2})\,d\beta$$

$$=\int_0^\infty exp\{-\lambda\sqrt{\beta^2+k^2}\}\cos(\beta\sqrt{t^2-\lambda^2})\,d\beta,$$

we have $\frac{\partial \varphi(\lambda,t,k)}{\partial \lambda}=0$. Therefore, $\varphi(\lambda,t,k)$ is independent of the variable number λ. (Namely, φ does not involve λ as a variable number.)

Consequently, $\varphi(\lambda,t,k)=\varphi(0,t,k)=\int_0^\infty \frac{\cos\beta t}{\sqrt{\beta^2+k^2}}d\beta=K_0(kt)$,

where K_0 is the 0-th order of the modified Bessel function. (Confer Watson, Bessel function, p. 185.)

Accordingly, we obtain

$$f(\lambda,z,k)=K_0(k\sqrt{\lambda^2+z^2}). \quad \cdots\cdots (19)$$

Replacing λ, k, and t in the formula (19) by $\lambda\sqrt{\frac{\nu_z}{\nu_x}}$, $\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}^{\frac{1}{2}}$, and $\sqrt{z^2+\lambda^2\frac{\nu_z}{\nu_x}}$ respectively, the required result I_1 is expressed by the following formula

$$I_1=\frac{1}{\sqrt{\frac{\nu_x}{\nu_z}}}K_0\left[\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}^{\frac{1}{2}}\sqrt{z^2+\lambda^2\frac{\nu_z}{\nu_x}}\right] \quad \cdots\cdots (20)$$

with the aid of the formulas (17), (18), and (19).

(B): Next, evaluate the underlying double integral:

$$I_2=\frac{2}{\pi}\int_0^\infty\int_0^\infty \frac{exp\left\{-\left\{\frac{\nu_x}{\nu_z}\alpha^2+\beta^2+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}\tau\right\}}{\frac{\nu_x}{\nu_z}\alpha^2+\beta^2+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}}$$

$$\times\cos\alpha\lambda\cos\beta z'\,d\alpha\,d\beta. \quad \cdots\cdots (21)$$

Differentiating the above I_2 with respect to τ,

we get $\frac{dI_2}{d\tau}=\frac{2}{\pi}\int_0^\infty\int_0^\infty -exp\left\{-\left\{\frac{\nu_x}{\nu_z}\alpha^2+\beta^2+\frac{(2n+1)^2\pi^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu}\right\}\tau\right\}$

$$\times\cos\alpha\lambda\cos\beta z'\,d\alpha\,d\beta.$$

Therefore, integrate the both-hand members of the above equation from 0 to τ in respect of τ, then we can obtain

$$I_2=-\frac{1}{2\sqrt{\frac{\nu_x}{\nu_z}}}\int_0^\tau \frac{exp\{-\left[\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}\tau+\frac{\lambda^2}{4\left(\frac{\nu_y}{\nu_z}\right)\tau}+\frac{z'^2}{4\tau}\right]\}}{\tau}d\tau+c \qquad \cdots\cdots(22)$$

where c is a arbitrary constant.

But since

$$|I_2|_{\tau=0}=c=\frac{2}{\pi}\int_0^\infty\int_0^\infty \frac{\cos\alpha\lambda\cos\beta z'}{\frac{\nu_x}{\nu_z}\alpha^2+\beta^2+\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}}d\alpha d\beta$$

$$=\frac{1}{\sqrt{\frac{\nu_x}{\nu_z}}}K_0\left[\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}^{\frac{1}{2}}\sqrt{z'^2+\lambda^2\frac{\nu_y}{\nu_z}}\right], \qquad \cdots\cdots(23)$$

from the formulas (20), (21), and (22),

the required result I_2 is given by the following formula:

$$I_2=\frac{1}{\sqrt{\frac{\nu_x}{\nu_z}}}K_0\left[\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}^{\frac{1}{2}}\sqrt{z'^2+\lambda^2\frac{\nu_z}{\nu_x}}\right]$$

$$-\frac{1}{2\sqrt{\frac{\nu_x}{\nu_z}}}\int_0^\tau \frac{exp\{-\left[\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}\tau+\frac{\lambda^2}{4\left(\frac{\nu_y}{\nu_z}\right)\tau}+\frac{z'^2}{4\tau}\right]\}}{\tau}d\tau. \qquad \cdots\cdots(24)$$

Thus, owing to the above sections ((A) and (B)), we have finished the preparations for evaluating the values of W_1, W_2, and $W=W_1-W_2$. Now, let us evaluate the values of those.

By applying (20) and (24) to the previous formulas (14), (15), and (16), we have the following solution for steady-state:

$$W_1=\frac{8T_0a^2exp\{i\phi\}}{A_z\pi^4}\sum_{n=1}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{(2n+1)^3}\int_{-x}^{l-x}\frac{1}{\sqrt{\frac{\nu_x}{\nu_z}}}K_0\left[\left\{\left(\frac{(2n+1)^2}{a^2}\right)\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}^{\frac{1}{2}}\sqrt{z'^2+\lambda^2\frac{\nu_z}{\nu_x}}\right]d\lambda$$

for $0\leqq x\leqq l$. ..... (25)

Next, we have the following solution for decayng state:

$$W_2=\frac{8T_0a^2exp\{i\phi\}}{A_z\pi^4}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{(2n+1)^3}\left[\int_{-x}^{l-x}\frac{1}{\sqrt{\frac{\nu_x}{\nu_y}}}K_0\left[\left\{\left(\frac{(2n+1)^2}{a^2}\right)\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}^{\frac{1}{2}}\sqrt{z'^2+\lambda^2\frac{\nu_z}{\nu_x}}\right]d\lambda\right.$$

$$\left.-\frac{1}{2\sqrt{\frac{\nu_x}{\nu_z}}}\int_0^{\tau}\int_{-x}^{l-x}\frac{exp\left\{-\left[\left\{\left(\frac{(2n+1)^2}{a^2}\right)\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}\tau+\frac{\lambda^2}{4\left(\frac{\nu_x}{\nu_z}\right)\tau}+\frac{z'^2}{4\tau}\right]\right\}}{\tau}d\lambda d\tau\right]$$

for $0\leqq x\leqq l$. ..... (26)

Consequently, the required solution for transient state is expressed by the underlying formulas.

$$W=W_1-W_2=\frac{4T_0a^2exp\{i\phi\}}{A_z\pi^4\sqrt{\frac{\nu_x}{\nu_z}}}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{(2n+1)^3}$$

$$\times\int_0^{\tau}\int_{-x}^{l-x}\frac{exp\left\{-\left[\left\{\left(\frac{(2n+1)^2}{a^2}\right)\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}\tau+\frac{\lambda^2}{4\left(\frac{\nu_x}{\nu_z}\right)\tau}+\frac{z'^2}{4\tau}\right]\right\}}{\tau}d\lambda d\tau.$$

But since $\int_{-x}^{l-x}exp\left\{-\frac{\lambda^2}{4\left(\frac{\nu_x}{\nu_z}\right)\tau}\right\}d\lambda=\sqrt{\pi}\sqrt{\frac{\nu_x}{\nu_z}}\tau\left[\Phi\left(\frac{l-x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)+\Phi\left(\frac{x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]$,

finally we have

$$W=\frac{4T_0a^2exp\{i\phi\}}{A_z\pi^3\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{(2n+1)^3}$$

$$\times\int_0^{\tau}\frac{\left[\Phi\left(\frac{l-x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)+\Phi\left(\frac{x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]exp\{-\left[\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}\tau+\frac{z'^2}{4\tau}\right]\}}{\sqrt{\tau}}d\tau$$

for $0\leqq x\leqq l$. ..... (27)

Similarly as treated above,

Steady state:

$$W_1=\frac{8T_0a^2exp\{i\phi\}}{A_z\pi^4}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{(2n+1)^3}\int_x^{l-x}\frac{1}{\sqrt{\frac{\nu_x}{\nu_z}}}K_0\left[\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}^{\frac{1}{2}}\sqrt{z'^2+\lambda^2\frac{\nu_z}{\nu_x}}\right]d\lambda$$

for $x<0$, ... (28)

$$W_1=\frac{8T_0a^2exp\{i\phi\}}{A_z\pi^4}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{(2n+1)^3}\int_{-(x-l)}^{\infty}\frac{1}{\sqrt{\frac{\nu_x}{\nu_z}}}K_0\left[\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}^{\frac{1}{2}}\sqrt{z'^2+\lambda^2\frac{\nu_z}{\nu_x}}\right]d\lambda$$

for $x>l$. ..... (29)

Transient state:

$$W=\frac{4T_0a^2exp\{i\phi\}}{A_z\pi^3\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{(2n+1)^3}$$

$$\times\int_0^{\tau}\frac{\left[\Phi\left(\frac{l-x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)-\Phi\left(\frac{x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]exp\{-\left[\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}\tau+\frac{z'^2}{4\tau}\right]\}d\tau}{\sqrt{\tau}}$$

for $x<0$, ... (30)

$$W=\frac{4T_0a^2exp\{i\phi\}}{A_z\pi^3\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{\sin\frac{(2n+1)\pi}{a}y}{(2n+1)^3}$$

$$\times\int_0^{\tau}\frac{\left[\Phi\left(\frac{x}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)-\Phi\left(\frac{x-l}{2\sqrt{\frac{\nu_x}{\nu_z}\tau}}\right)\right]exp\{-\left[\left\{\frac{(2n+1)^2}{a^2}\frac{\nu_y}{\nu_z}+i\frac{2\bar{\omega}}{\nu_z}\right\}\tau+\frac{z'^2}{4\tau}\right]\}d\tau}{\sqrt{\tau}}$$

for $x>l$. ..... (31)

Appendix

SHORT NOTE ON THE THEORY OF DRIFT CURRENTS DRIVEN BY WIND IN AN ENCLOSED INFINITELY DEEP OCEAN

(*The case of the variation of the Coriolis parameter with latitude*)

§ 1. Introduction

As a result of the recent development of Sverdrup, Stockman, Stommel, Reid, and Hidaka, the subject of drift currents in an enclosed ocean has made two very notable advances. On the one hand, both the foundation and the application of the theory of drift currents have been simplified by introducing the mass transport stream function into the equation of motion used. On the other, the variation of the Coriolis parameter with latitude which brings the westward intensification has at least partially occupied the more important standpoint in this theory.

In the reports so far published by several authors, emphasis has naturally been placed on the second of these advances: the magnitude of the first has been somewhat obscured by the fact that the mass transport method have been applied to problems which were beyond the range of difficulties of the mathematical treatment of the older theory (the treatments of Ekman and others), so that the simplicity of the method hid the complexity of the problem. (Because, the mass transport method does not permit us to peep at the vertical structure of ocean currents.) Furthermore, the theory of Stommel and Munk differs so profoundly in its formulation from that of Ekman and others, and so little of it has been published, that its particular advantage have not hitherto been made clear to us in the general circulation of an enclosed ocean. Though the advantage propounded by Stommel and Munk are simplicity and ease of application, the theory of them does not permit us to peep at the vertical structure of ocean currents.

The present paper aims to show how the variation of the current velocity components in the horizontal direction as well as in the vertical can be expressed in a rotating rectangular ocean, when the currents are produced by wind stress.

The main results of the paper will be general formulas from which a picture of ocean currents produced by wind stress can be obtained, treating the problem of drift currents in an infinitely deep rectangular ocean (as a special case) by taking account of the horizontal and vertical mixing as well as the variation of the Coriolis parameter with latitude, to some desired order of approeimation.

The detail of this results will, however, be reserved for the next paper.

§ 2. Summary of our treatment

In the problems so far treated by the author (Chapters 1 and 2), they were discussed under the assuption that the Coriolis parameter does not vary with latitude. But, in the ocean which is of a wide range, such as Pacific Ocean and Atlantic Ocean, when the variation of the Coriolis parameter with latitune is taken into account in the equation of motion used, we can instinctly expect the correct results for these problems (especially in the case of an enclosed ocean).

The importance of the above conception was firstly admitted and discussed by Dr. Stommel (1948), and consequently the westward intensification was shown as the effect of the variation of the Coriolis parameter with latitude. Subsequently, Dr. Muhk (1950) published the more preciser theory which was considered as the generalization of the Stommel's treatment by taking the terms of the horizontal and vertical mixing into account simultaneoesly. The same treatment as this was also propounded by Dr. Hidaka. As stated in the previous chapter, however, the mass transport method which is applied to this theory by Munk does not permit us to peep at the vertical structure of the oceanic current.

In this respect it is desirable to treat these problems by using the velocity components themselves instead of the mass transport. This short note is the result of the effort in this direction.

For the sake of simplification, the case of the drift currents in an infinitely deep rectangular ocean is treated in the present paper, and the mode of the distribution of wind-stress given in this case is instinctly the same one as given by Stommel. (Accordingly, the region in our treatment is restricted to the case of the ocean in the north-hemisphere only.) Let us evaluate the analytical solution for our problem in the underlying discussion.

In the method of the solution of the equations of motion treated by Munk, introducing the integrated mass transport stream function ψ into the simultaneous differential equations of motion and performing the curl operation to eliminate the integrated pressure function P from them, we have the differential equation in which only ψ is considered as an unknown-function, and ψ is evaluated to be satisfied by the boundary conditions involving the appropriate physical meaning that the current velocity is zero along the coastlines. We also aim to evaluate the required current velocity under the boundary condition that it is zero along the coast-lines, after Munk. But it

is doubtful whether the solution of the simultaneous differential equations by Munk is always correct mathematically, or not. The reason is as follows.

In general, on the boundary value problem of the differential equation, if the order of the differential equation is raised by performing the curl operation, then the new boundary conditions, whose numbers are identical with those of the order raised, must be added to the previously given conditions. And the added conditions should be mathematically estimated without considering the physical meanings (except the case when the boundary conditions derived from the mathematical treatments are identical with those from the physical meanings).

From the point of view menthioned above, it appears to be unallowable that the Munk's treatment is always recognized reasonable.

So, it is necessary that the original differential equation of motion is directly solved without performing the curl operation. But the detail of this treatment will be reserved for the near future. In the present paper, let us consider the problems of the drift currents in an infinitely deep rectangular

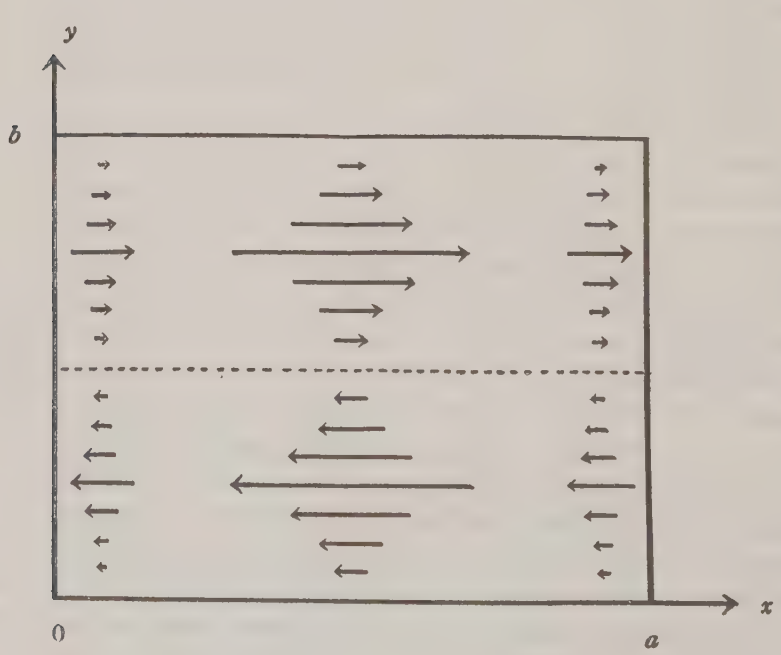

Fig. 6 Distribution of wind stress

ocean as the simplest case. As shown in Fig. 6, let the x-axis point eastward along the equator, the y-axis northward, and the z-axis vertically upwards from a level surface (assumed plane) just beneath the sea surface. (Where for the sake of simplification let us consider only the case of the north hemispher.)

The shores of the ocean are at $x=0$, a and $y=0$, b, and we shall consider the ocean as a homogeneous one of infinite depth. (See Fig. 6.)

Denote the notations being the same as those of our previous paragraph, then the required currents will be represented by the solution of the following equations:

$$\frac{\partial W}{\partial t}=\nu_x\frac{\partial^2 W}{\partial x^2}+\nu_y\frac{\partial^2 W}{\partial y^2}+\nu_z\frac{\partial^2 W}{\partial z^2}-i\beta y W \qquad \cdots\cdots(1)$$

(where $\beta=\frac{df}{dy}$ is the rate of change northward of twice the vertical component of the earth's angular velocity, and $f=2\omega\sin\varphi$: φ=the latitude under consideration, ω=the earth's angular velocity),

and under the initial and boundary conditions:

$$|W|_{t=0}=0 \qquad \text{when } t=0\ , \qquad \cdots\cdots(2)$$

$$\left.\begin{aligned}|W|_{x=0}&=0 \qquad |W|_{x=a}=0\\ |W|_{y=0}&=0 \qquad |W|_{y=b}=0\end{aligned}\right\}\ \text{along the coast-lines}\ , \qquad \cdots\cdots(3),\ \cdots\cdots(4)$$

$$\left|A_z\frac{\partial W}{\partial z}\right|_{z=0}=T \quad \text{at the sea surface } (z=0)\ , \qquad \cdots\cdots(5)$$

and simultaneously

$$\left.\begin{aligned}\left|\frac{\partial W}{\partial z}\right|_{z=-\infty}&=0\\ |W|_{z=-\infty}&=0\end{aligned}\right\}\ \text{at the sea bottom } (z=-\infty)\ . \qquad \cdots\cdots(6)$$

The wind over the ocean are the Trade over the equatorial half of the rectangular basion, and Prevailing westeries over the poleward half. And expression for the wind stress acting upon a column of unit-cross-section must include this independence upon y.

In our case a functional form of the wind stress is taken as

$$T=T_0 x(x-a)\sin\frac{2\pi}{b}y \ . \qquad \cdots\cdots(7)$$

In order to solve the equation (1), we shall use Stokes method. Let W be expressed by Fourier's sine series $(0<x<a)$, then we have

$$W=\frac{2}{a}\sum_{s=1}^{\infty}\Phi_s\sin\frac{s\pi}{a}x \qquad \cdots\cdots(8)$$

$$\text{where}\quad \Phi_s=\int_0^a W(\lambda,y,z,t)\sin\frac{s\pi}{a}\lambda d\lambda \ . \qquad \cdots\cdots(9)$$

Accordingly, $$\frac{\partial^2 W}{\partial x^2}=-\frac{2}{a}\sum_{s=1}^{\infty}\left(\frac{s\pi}{a}\right)^2\Phi_s\sin\frac{s\pi}{a}x$$

with the aid of the formula (3).

And also $$\frac{\partial^2 W}{\partial y^2}=\frac{2}{a}\sum_{s=1}^{\infty}\frac{\partial^2\Phi_s}{\partial y^2}\sin\frac{s\pi}{a}x \ ,$$

$$\frac{\partial^2 W}{\partial z^2}=\frac{2}{a}\sum_{s=1}^{\infty}\frac{\partial^2\Phi_s}{\partial z^2}\sin\frac{s\pi}{a}x \ ,$$

$$\frac{\partial W}{\partial t}=\frac{2}{a}\sum_{s=1}^{\infty}\frac{\partial\Phi_s}{\partial t}\sin\frac{s\pi}{a}x \ .$$

Moreover, we may expand $T(x, y)$ in Fourier's sine series such as

$$T=\frac{2}{a}\sum_{s=1}^{\infty}T_s(\lambda)\sin\frac{s\pi}{a}x \ ,$$

$$\text{where}\quad T_s=\int_0^a T\sin\frac{s\pi}{a}\lambda d\lambda \ .$$

Substituting this above formulas into the equations (1) and equating the factor of $\sin\frac{s\pi}{a}x$, we have the following equation:

$$\frac{\partial\Phi_s}{\partial t}=-\nu_x\left(\frac{s\pi}{a}\right)^2\Phi_s+\nu_y\frac{\partial^2\Phi_s}{\partial y^2}+\nu_z\frac{\partial^2\Phi_s}{\partial z^2}-i\beta y\Phi_s \ . \qquad \cdots\cdots(10)$$

Corresponding to (10), the conditions (2), (4), (5), and (6) become as follows.

$$|\Phi_s|_{t=0} = 0 \ , \qquad \cdots\cdots (11)$$

$$|\Phi_s|_{y=0} = 0 \ , \quad |\Phi_s|_{y=b} = 0 \ , \qquad \cdots\cdots (12)$$

$$\left| A_z \frac{\partial \Phi_s}{\partial z} \right|_{z=0} = T_s \ , \qquad \cdots\cdots (13)$$

$$\left| \frac{\partial \Phi_s}{\partial z} \right|_{z=-\infty} = 0 \quad \text{and simultaneously} \quad |\Phi_s|_{z=-\infty} = 0 \ , \qquad \cdots\cdots (14)$$

Next, moreover express Φ_s with Fourier's Integral, then

$$\Phi_s = \frac{2}{\pi} \int_0^\infty U(\alpha) \cos \alpha z' d\alpha \qquad \cdots\cdots (15)$$

where $$U = \int_{-\infty}^0 \Phi(\xi) \cos \alpha \xi d\xi \qquad \cdots\cdots (16)$$

and $z = -z'$.

Similarly,
$$\frac{\partial^2 \Phi_s}{\partial z^2} = \frac{2}{\pi} \int_0^\infty d\alpha \int_{-\infty}^0 \frac{\partial^2 \Phi_s}{\partial \xi^2} \cos \alpha z' \cos \alpha \xi d\xi$$

$$= \frac{2}{\pi} \int_0^\infty \left\{ \frac{T_s}{A_z} - \alpha^2 U(\alpha) \right\} \cos \beta z' d\alpha$$

by the help of the formulas (13) and (14).

And also
$$\frac{\partial \Phi_s}{\partial t} = \frac{2}{\pi} \int_0^\infty d\alpha \int_{-\infty}^0 \frac{\partial U}{\partial t} \cos \alpha z' d\alpha \ ,$$

$$\frac{\partial^2 \Phi_s}{\partial y^2} = \frac{2}{\pi} \int_0^\infty \frac{\partial^2 U}{\partial y^2} \cos \alpha z' dz \ .$$

Substituting these above obtained formulas into (10), we have

$$\frac{\partial U}{\partial \tau} = \frac{\partial^2 U}{\partial y^2} - \left\{ \frac{\nu_z}{\nu_y} \left(\frac{s\pi}{a} \right)^2 + \alpha^2 + i\beta y \right\} U + \frac{T_s}{A_z} \ , \qquad \cdots\cdots (17)$$

where $\tau = \nu_z t$.

Corresponding to (17), the conditions (11) and (12) become

$$|U|_{\tau=0}=0 \quad \cdots\cdots(18) \quad \text{and} \quad |U|_{y=0}=0, \quad |U|_{y=b}=0. \quad \cdots\cdots(19)$$

Now, divide U into two parts: a steady part U_1 and a varying part U_2 such that

$$U=U_1-U_2. \quad \cdots\cdots(20)$$

Steady part U_1. This part must satisfy the equations

$$0=\frac{\partial^2 U_1}{\partial y^2}-\left\{\frac{\nu_x}{\nu_z}\left(\frac{s\pi}{a}\right)^2+\alpha^2+i\beta y\right\}U_1+\frac{T_s}{A_z}, \quad \cdots\cdots(21)$$

$$|U_1|_{y=0}=0 \quad , \quad |U_1|_{y=b}=0. \quad \cdots\cdots(22)$$

The general solution of the eqaution (21) is obviously

$$U_1=Af_1(y)+Bf_2(y)+f_1(y)\int\frac{\frac{T_s}{A_z}f_2(y)\,dy}{f_1(y)f_2'(y)-f_1'(y)f_2(y)}-f_2(y)\int\frac{\frac{T_s}{A_z}f_1(y)\,dy}{f_1(y)f_2'(y)-f_1'(y)f_2(y)},$$

where A and B are arbitrary constants which are determined under the conditions (22), and also

$$f_1(y)=\left\{i\beta y+\frac{\nu_x}{\nu_z}\left(\frac{s\pi}{a}\right)^2+\alpha^2\right\}^{\frac{1}{2}}J_{\frac{1}{3}}\left[\frac{2}{3i\beta}\left\{i\beta y+\frac{\nu_x}{\nu_z}\left(\frac{s\pi}{a}\right)^2+\alpha^2\right\}^{\frac{3}{2}}\right],$$

$$f_2(y)=\left\{i\beta y+\frac{\nu_x}{\nu_z}\left(\frac{s\pi}{a}\right)^2+\alpha^2\right\}^{\frac{1}{2}}Y_{\frac{1}{3}}\left[\frac{2}{3i\beta}\left\{i\beta y+\frac{\nu_x}{\nu_z}\left(\frac{s\pi}{a}\right)^2+\alpha^2\right\}^{\frac{3}{2}}\right].$$

Varying part U_2. This part must satisfy the equations

$$\frac{\partial U_2}{\partial\tau}=\frac{\partial^2 U_2}{\partial y^2}-\left\{\frac{\nu_x}{\nu_z}\left(\frac{s\pi}{a}\right)^2+\alpha^2+i\beta y\right\}U_2, \quad \cdots\cdots(23)$$

$$|U_2|_{\tau=0}=U_1, \quad \cdots\cdots(24)$$

$$|U_2|_{y=0}=0 \quad , \quad |U_2|_{y=b}=0. \quad \cdots\cdots(25)$$

The general solution of the equation (23) is obviously

$$U_2=exp\{-\gamma^2\tau\}\,[c_1\chi_1(y,\gamma)+c_2\chi_2(y,\gamma)]\,,$$

where c_1, c_2, and γ are numbers which are determined under the conditions (24) and (25). Moreover,

$$\chi_1(y,\gamma)=\left\{i\beta y+\frac{\nu_x}{\nu_z}\left(\frac{s\pi}{a}\right)^2+\alpha^2-\gamma^2\right\}^{\frac{1}{2}}J_{\frac{1}{3}}\left[\frac{2}{3i\beta}\left\{i\beta y+\frac{\nu_x}{\nu_z}\left(\frac{s\pi}{a}\right)^2+\alpha^2-\gamma^2\right\}^{\frac{3}{2}}\right],$$

$$\chi_2(y,\gamma)=\left\{i\beta y+\frac{\nu_x}{\nu_z}\left(\frac{s\pi}{a}\right)^2+\alpha^2-\gamma^2\right\}^{\frac{1}{2}}Y_{\frac{1}{3}}\left[\frac{2}{3i\beta}\left\{i\beta y+\frac{\nu_x}{\nu_z}\left(\frac{s\pi}{a}\right)^2+\alpha^2-\gamma^2\right\}^{\frac{3}{2}}\right].$$

The detail of these analysis will be published in the near future.

Chapter 3

TRANSIENT STATE OF DRIFT CURRENTS IN A ROTATING CANAL

(*The case of finite depth*)

§ 1. Introduction

In Chapter 1 we have already treated the case of steady drift currents in a rotating canal of finite depth. And in Chapter 2 only the case of the transient state of drift currents in an infinitely deep ocean was discussed. Accordingly, the case of the transient state of drift currents in an ocean of finite depth is still reserved. However, as the treatment of the case of the ocean which is boundless in horizontal extent is mathematically difficut, we aim to deal with the case of a rotating canal. As to the theory of this case, it has been already propounded by Horrock (1927) and T. Nomitsu (1934). But in their treatment the terms of the horizontal mixing in the equation of motion was entirely neglected, though Prof. Nomitsu of course pointed out the necessity of this term at that time. Therefore, the horizontal current velocity obtained by him does not vanish along the coast-lines themselves, provided the wind stress is not zero along these ones.

To avoid this defect, as stated before, we are obliged to introduce the term of the horizontal mixing into the equation of motion used. In the underlying discussion we treat the case of the transient state of current and surface slope in a rotating canal of finite depth generated by wind action, taking the terms of the horizontal and vertical mixing into account. (Where, in our case the variation of the Coriolis parameter with latitude is neglected.) And moreover, our present object is to ascertain how the slope of sea level gradually develop as the effect of the accumlation of waters driven by winds. The components of the tangential stress of wind on the sea-surface may be given by any functions of y, which now may be denoted by $T(y)$, where $T=T_r+iT_i$, $i=\sqrt{-1}$; the indices r and i mean real and imaginary parts respectively.

If we consider two infinitely long straight coasts, it is obvious that the slope is always formed in the direction perpendicular to the coasts. Then the said slope is obtained as the solution of integral equation presented in the underlying paragraph. However, we can easily solve the differential equations resulted from performing the appropriative transformation to this integral equation. The case of the variation of the Coriolis parameter with latitude will be reserved for the near future.

§ 2. The equation of motion and boundary condition

First, consider a rotating canal of uniform depth bounded by two

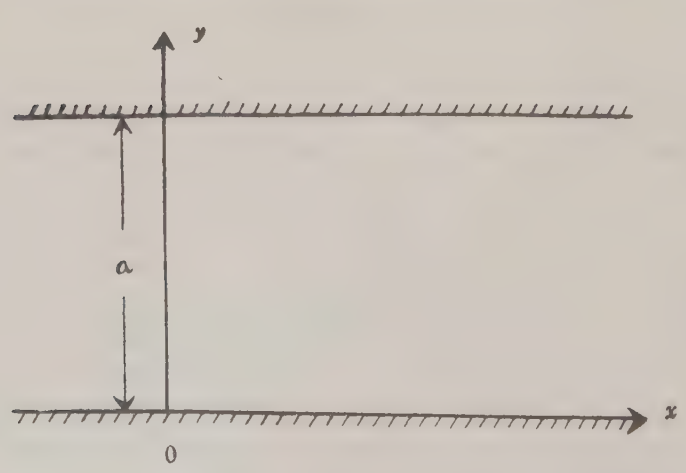

Fig. 7

infinite parallel straight coasts distance a apart and suppose that a wind (whose stress is $T(y)=T_r+iT_i$) suddenly begins to blow over the sea which is initially at rest.

Then the dynamical equations of the transient state of ocean currents, taking both the vertical and horizontal mixing into account and neglecting the non-linear terms, are

$$\frac{\partial W}{\partial t}=\nu_y\frac{\partial^2 W}{\partial y^2}+\nu_z\frac{\partial^2 W}{\partial z^2}-i2\bar{\omega}\,W-ig\gamma_y\,, \qquad \cdots\cdots(1)$$

where $W=u+iv$; u and v are components of the current velocity in x and y direction (where the x axis parallel to the coasts, the y axis perpendiculary to the coasts), γ_y a component of the slope of sea level $\left(=-\frac{\partial\zeta}{\partial y}\right.$, ζ: the elevation of sea water level) in y direction, ν_y and ν_z the horizontal and vertical coefficients of kinematic eddy viscocity, and t a time, while $\bar{\omega}=\omega\sin\varphi$, and ω the angular velocity of the earth, φ the geographical latitude. The axis of z is taken positive downward, the origin being placed on the undisturbed mean sea level. (See Fig. 7)

The boundary conditions to be satisfied on the surface ($z=0$) and at the bottom ($z=h$) are

$$\left.\begin{aligned}\mu\left|\frac{\partial W}{\partial z}\right|_{z=0}&=-T(y)\\ W|_{z=h}&=0\,.\end{aligned}\right\} \qquad \cdots\cdots(2)$$

Here the coefficient of eddy viscosity μ is supposed to be constants. The conditions to be satisfied along the coasts are also necessary. These are simply that there is no water flow across and along these coasts. If the coasts consist of vertical cliffs, we have

$$|W|_{y=0}=|W|_{y=a}=0 \qquad \text{along the coast-lines.} \qquad \cdots\cdots(3)$$

In addition to the dynamical equation (1) we shall have the equation of continuity. Namely, $\frac{\partial\zeta}{\partial t}=-Im\left[\frac{\partial S}{\partial y}\right]$, where $S=\int_0^h W dz$, and Im means the imaginary part.

But since $\gamma_y=-\frac{\partial\zeta}{\partial y}$, the above equation becomes

$$\frac{\partial\gamma_y}{\partial t}=Im\left[\frac{\partial^2 S}{\partial y^2}\right]. \qquad \cdots\cdots(4)$$

And the initial conditions are

$$|W|_{t=0}=0, \quad \cdots\cdots(5) \qquad |\gamma_y|_{t=0}=0. \quad \cdots\cdots(6)$$

§ 3. The solution of differential equation

In order to solve the equation (1), we shall use Stokes method. Let W be expressed by Fonrier's sine series, then

$$W=\frac{2}{a}\sum_{s=1}^{\infty}\Phi_s \sin\frac{s\pi}{a}y, \quad \cdots\cdots(7)$$

where $$\Phi_s=\int_0^a W \sin\frac{s\pi}{a}\lambda d\lambda. \quad \cdots\cdots(8)$$

Similarly, $$\gamma_y=\frac{2}{a}\sum_{s=1}^{\infty}\Gamma_s \sin\frac{s\pi}{a}\lambda d\lambda, \quad \cdots\cdots(9)$$

where, $$\Gamma_s=\int_0^a \gamma_y \sin\frac{s\pi}{a}\lambda d\lambda. \quad \cdots\cdots(10)$$

Moreover, $$\frac{\partial^2 W}{\partial y^2}=-\frac{2}{a}\sum_{s=1}^{\infty}\left(\frac{s\pi}{a}\right)^2\Phi_s \sin\frac{s\pi}{a}y \quad \cdots\cdots(11)$$

by the help of the formula (3).

And $$T(y)=\frac{2}{a}\sum_{s=1}^{\infty}T_s \sin\frac{s\pi}{a}y, \quad \cdots\cdots(12)$$

where $$T_s=\int_0^a T \sin\frac{s\pi}{a}\lambda d\lambda. \quad \cdots\cdots(13)$$

Accordingly, introducing the formulas from (7) to (11) into the formula (1), equate the factor of $\sin\frac{s\pi}{a}y$. Then we have

$$\frac{\partial\Phi_s}{\partial t}=\nu_z\frac{\partial^2\Phi_s}{\partial z^2}-\left\{\nu_y\left(\frac{s\pi}{a}\right)^2+i\,2\bar{\omega}\right\}\Phi_s-ig\Gamma_s. \quad \cdots\cdots(14)$$

Corresponding to (14), the initial and boundary conditions are

$$\mu\left|\frac{\partial\Phi_s}{\partial z}\right|_{z=0}=-T_s, \quad \cdots\cdots(15) \qquad |\Phi_s|_{z=h}=0, \quad \cdots\cdots(16)$$

$$|\Phi_s|_{t=0}=0, \quad \cdots\cdots(17) \qquad |\Gamma_s|_{t=0}=0. \quad \cdots\cdots(18)$$

Accordingly, the equation (14) becomes the following expression.

$$\frac{\partial \Phi_s}{\partial t}=\nu_z \frac{\partial^2 \Phi_s}{\partial z^2}-\left\{\nu_y\left(\frac{s\pi}{a}\right)^2+i\,2\bar{\omega}\right\}\Phi_s-igT_s\,. \qquad \cdots\cdots (14')$$

Now, when the wind does not vary with the time, the problem may be reduced to two simpler cases, one of these being the case of pure drift currents and the other case of pure slope currents.

That is, we can divide Φ_s into two parts such that $\Phi_s=\Phi_{sT}-\Phi_{s\Gamma}$, where Φ_{sT} corresponds to the pure drift currents primarily caused by only the wind stress T, and $\Phi_{s\Gamma}$ to the pure slope currents produced by the surface slope Γ_s which is secondarily induced by the primary currents.

Primary current Φ_{sT}. The differential equation and the initial and boundary conditions of Φ_{sT} are as follows.

$$\frac{\partial \Phi_{sT}}{\partial t}=\nu_z \frac{\partial^2 \Phi_{sT}}{\partial z^2}\left\{\nu_y\left(\frac{s\pi}{a}\right)^2+i\,2\bar{\omega}\right\}\Phi_{sT}, \qquad \cdots\cdots (19)$$

$$\Phi_{sT}=0 \qquad \cdots\cdots (20) \quad , \qquad \mu\left|\frac{\partial \Phi_{sT}}{\partial z}\right|=-T_s \quad , \qquad \cdots\cdots (21)$$

$$|\Phi_{sT}|=0 \quad . \qquad \cdots\cdots (22)$$

Then, Φ_{sT} to be satisfied by these formulas from (19) to (22) is given by the following expression.

$$\Phi_{sT}=\sum_{n=0}^{\infty}\frac{2T_s}{\rho h}\cos\beta_n z\int_0^t exp\left\{-\left\{\nu_z\beta_n^2+\nu_y\left(\frac{s\pi}{a}\right)^2+i\,2\bar{\omega}\right\}t\right\}dt\,. \qquad \cdots\cdots (23)$$

$$\text{where} \quad \beta_n=\frac{(2n+1)}{2h}\pi.$$

Secondary current $\Phi_{s\Gamma}$ .The differential equation and the initial and boundary conditions of $\Phi_{s\Gamma}$ are as follows.

$$\frac{\partial \Phi_{s\Gamma}}{\partial t}=\nu_z \frac{\partial^2 \Phi_{s\Gamma}}{\partial z^2}-\left\{\nu_y\left(\frac{s\pi}{a}\right)^2+i\,2\bar{\omega}\right\}\Phi_{s\Gamma}-igT_s, \qquad \cdots\cdots (24)$$

$$|\Phi_{s\Gamma}|=0, \qquad \cdots\cdots (25) \qquad \left|\frac{\partial \Phi_{s\Gamma}}{\partial z}\right|=0, \qquad \cdots\cdots (26) \qquad |\Phi_{s\Gamma}|=0. \qquad \cdots\cdots (27)$$

The solution of the above differential equation is as follows.

$$\Phi_{s\Gamma}=\frac{i2g}{h}\sum_{n=0}^{\infty}\frac{(-1)^n}{\beta_n}\cos\beta_n z\int_0^t \Gamma_s(\tau)\,exp\{-\left\{\nu_z\beta_n{}^2+\nu_y\left(\frac{s\pi}{a}\right)^2+i\,2\bar{\omega}\right\}(t-\tau)\}d\tau. \quad \cdots\cdots (28)$$

Therefore,

$$\Phi_s=\Phi_{sT}+\Phi_{s\Gamma}=\frac{2T_s}{\rho h}\sum_{n=0}^{\infty}\cos\beta_n z\int_0^t exp\{-\left\{\nu_z\beta_n{}^2+\nu_y\left(\frac{s\pi}{a}\right)^2+i2\bar{\omega}\right\}t\}\,dt$$

$$+\frac{i2g}{h}\sum_{n=0}^{\infty}\frac{(-1)^n}{\beta_n}\cos\beta_n z\int_0^t \Gamma_s(\tau)\,exp\{-\left\{\nu_z\beta_n{}^2+\nu_y\left(\frac{s\pi}{a}\right)^2+i2\bar{\omega}\right\}(t-\tau)\}\,d\tau. \quad \cdots\cdots (29)$$

Accordingly, $$W=\frac{2}{a}\sum_{s=1}^{\infty}\sum_{n=0}^{\infty}\sin\frac{s\pi}{a}y\left[\Phi_{sT}+\Phi_{s\Gamma}\right]. \quad \cdots\cdots (30)$$

Next, applying the equation of continuity (4), let us evaluate the value of Γ_s. Namely,

$$Im\left[\frac{\partial^2 S}{\partial y^2}\right]=Im\left[\frac{\partial^2}{\partial y^2}\int_0^h W dz\right]$$

$$=\frac{2}{a}\sum_{s=1}^{\infty}\sum_{n=0}^{\infty}\sin\frac{s\pi}{a}y\left[-\left(\frac{s\pi}{a}\right)^2\right]\left[\frac{2}{\rho h}\frac{(-1)^n}{\beta_n}\right.$$

$$\times\left\{T_{si}\int_0^t exp\{-\left\{\nu_z\beta_n{}^2+\nu_y\left(\frac{s\pi}{a}\right)^2\right\}t\}\cos 2\bar{\omega}t\,dt\right.$$

$$\left.+T_{sr}\int_0^t exp\{-\left\{\nu_z\beta_n{}^2+\nu_y\left(\frac{s\pi}{a}\right)^2\right\}t\}\sin 2\bar{\omega}t\,dt\right\}$$

$$\left.+\frac{2g}{h}\frac{(-1)^n}{\beta_n{}^2}\int_0^t \Gamma(\tau)\,exp\{-\left\{\nu_z\beta_n{}^2+\nu_y\left(\frac{s\pi}{a}\right)^2\right\}(t-\tau)\}\cos 2\bar{\omega}(t-\tau)d\tau\right], \quad \cdots\cdots (31)$$

and $$\frac{\partial\gamma_y}{\partial t}=\frac{2}{a}\sum_{s=1}^{\infty}\frac{\partial\Gamma_s}{\partial t}\sin\frac{s\pi}{a}y. \quad \cdots\cdots (32)$$

Therefore, applying the formulas (31) and (32), we have

$$\frac{\partial \Gamma_s}{\partial t}=-\left(\frac{s\pi}{a}\right)^2\frac{2(-1)^n}{\rho h\beta_n}\left[\frac{p_{n,s}T_{st}+qT_{sr}}{p_{n,s}^2+q^2}+\sqrt{p_{n,s}^2+q^2}\,exp\{-p_{n,s}t\}\left\{\sin\left(qt+\varphi_{n,s}+\frac{\pi}{2}\right)\right.\right.$$

$$\left.\left.+\sin(qt+\varphi_{n,s})\right\}+\frac{2g}{h}\frac{1}{\beta_n^2}\int_0^t \Gamma_s(\tau)\,exp\{-p_{n,s}(t-\tau)\}\cos q(t-\tau)\,d\tau\right], \quad \cdots\cdots(33)$$

where $\tan\varphi_{n,s}=\dfrac{q}{p_{n,s}}$, $p_{n,s}=\nu_z\beta_n^2+\nu_y\left(\dfrac{s\pi}{a}\right)^2$, $q=2\bar{\omega}$.

In order to solve (35), put $\Gamma_s(t)=exp\{-p_{n,s}t\}X_{n,s}(t)$. $\cdots\cdots(34)$

Then, differentiating the equation obtained by introducing (34) into (33) in respect of t twice and combining the result with the above equation, we have

$$\frac{d^3X_{n,s}}{dt^3}-p_{n,s}\frac{d^2X_{n,s}}{dt^2}+\left(q^2+\frac{s^2\pi^2}{a^2}\frac{2g}{h}\frac{1}{\beta_n^2}\right)\frac{dX_{n,s}}{dt}-p_{n,s}q^2X_{n,s}$$

$$=-\left(\frac{s\pi}{a}\right)^2\frac{2(-1)^n}{\rho h\beta_n}(p_{n,s}T_{st}+qT_{sr})\,exp\{p_{n,s}t\}. \quad \cdots\cdots(35)$$

Where, the initial conditions are $|X_{n,s}|_{t=0}=0$, $\cdots\cdots(36)$

$$\left|\frac{dX_{n,s}}{dt}\right|_{t=0}=-\left(\frac{s\pi}{a}\right)^2\frac{2(-1)^n}{\rho h\beta_n}\left[\frac{p_{n,s}T_{st}+qT_{sr}}{p_{n,s}^2+q^2}+\sqrt{p_{n,s}^2+q^2}\left\{\sin\left(\varphi_{n,s}+\frac{\pi}{2}\right)+\sin\varphi_{n,s}\right\}\right],$$

$\cdots\cdots(37)$

$$\left|\frac{d^2X_{n,s}}{dt^2}\right|_{t=0}=-\left(\frac{s\pi}{a}\right)^2\frac{2(-1)^n}{\rho h\beta_n}\left[\frac{2(p_{n,s}T_{st}+qT_{sr})}{p_{n,s}^2+q^2}+(p_{n,s}^2+q^2)\left\{\sin\left(2\varphi_{n,s}+\frac{\pi}{2}\right)+\sin 2\varphi_{n,s}\right\}\right].$$

$\cdots\cdots(38)$

The solution of the above equation is obviously

$$X_{n,s}=A_{n,s}\,exp\{p_{n,s}t\}+exp\left\{\tfrac{1}{3}p_{n,s}t\right\}(B_{n,s,1}\,exp\{\sigma'_{n,s}t\}+B_{n,s,2}\,exp\{\sigma''_{n,s}t\}$$

$$+B_{n,s,3}\,exp\{\sigma'''_{n,s}t\}), \quad \cdots\cdots(39)$$

where $B_{n,s,1}$ $B_{n,s,2}$, and $B_{n,s,3}$ are integration constants, and indices $\sigma'_{n,s}$, $\sigma''_{n,s}$, $\sigma'''_{n,s}$ are the following roots of cardin's cubic equation's type

$$\sigma^3_{n,s}+3C\,\sigma_{n,s}+2D=0. \quad \text{(C and D are any constant.)}$$

Above three integration constants can be determined by three initial relations (36), (37), and (38).

Thus, $X_{n,s}$ being determined completely, the elemental slope Γ_s will be given by

$$\Gamma_s=\bar{\Gamma}_s+exp\{-\tfrac{2}{3}p_{n,s}t\}(B_{n,,1}\,exp\{\sigma'_{n,s}t\}+B_{n,s,2}\,exp\{\sigma''_{n,s}t\}+B_{n,s,3}\,exp\{\sigma'''_{n,s}t\}), \quad \cdots\cdots(40)$$

where $\bar{\Gamma}_s$ means the final steady value of $\bar{\Gamma}_s$.

Next, applying the formula (40) to the formulas (9), (28), and (30), we can obtain W and Γ.

I am sorry I cannot show the complete results before the printing of this paper. But it will be published in the near future.

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REFERENCE

EKMAN, V. WALFRID, 1905: On the influence of the earth's rotation on ocean-currents. Arkiv. Astr. Matem. Och Fys. 3.

PROUDMAN, J. and DOODSON, A. T., 1924: Time-relations in meteorological effects on the sea.
Proc. math. Soc. London, Ser. 2, Bd. 24, Part 2.

HIDAKA, K., 1933: Non-stationary ocean-currents. Mem. Imp. Mar. Obseratory, Kōbe, 5, 255.

KOENUMA, K., 1933: Umi to Sora. Vol. 13, 67. (in Japanese)

NOMITSU, T., 1933: A theory of the rising stage of drift current in the ocean I, II, III. Mem. of Kyōto Imp. Univ. vol. XVI, No. 2, No. 4, No. 5. On the development of the slope current and the barometric current in the ocean. I. II. Mem. of Kyōto Imp. Univ. vol. XVI, No. 3, No. 5.

NOMITSU, T., 1934: Coast effect upon the ocean current and the sea level. I, II. Mem. of Kyōto Imp. Univ. vol. XVII, No. 3. No. 5.

TAKEGAMI, T., 1934: The boundary value problem of the wind current in a lake or a sea. Mem. of Kyōto Imp. Univ. vol. XVII, No. 5.

ROSSBY, C. G., 1936: Dynamics of steady ocean currents in the light of experimental fluid mechanics. Pap. Phys. Ocean. Meteor., Mass. Inst. Tech. Woods Hole Ocean. Instn. 5, No. 1.

HIDAKA, K., 1939: Wind circulation of water and the inertia terms, Memoirs of the Imperial Marine Observatory. Kōbe, vol. VII. No. 1.

OKAMOTO, M., 1942: Chikyubutsuri, vol. 7, No. 1. (in Japanese)

STOCKMAN, W. B., 1946: Equations of a field of total flow induced by the wind in a non-homogeneous sea. Computes Rendus (Dklady) d 1' Acad. Sciences de 1' URSS 54 (6): 403–406.

FJELDSTAD, J. E., 1946: Stationary Currents is heterogeneous water. Archiv Math. Naturrid., 48, No. 6.

SVERDRUP, H. U., 1947: Wind-driven currents in a baroclinic ocean; with application to the equatorial currents of the Eastern Pacific. Nat. Acad. Sci. Proc. 33 (11); 318–326.

REID, ROBERT O., 1948: The equatorial currents of the eastern Pacific as mentioned by the stress of the winds. J. Mar. Res. 7 (2).

STOMMEL, H., 1948: Westward intensification of wind-driven ocean currents. Amer. Geoph. Union Trans., 29 (2).

MUNK, W. H., 1950: On the wind-driven ocean circulation. Journal of Meteorology, 7 (2), 79–93.

HIDAKA, K., 1949: Mass transport in ocean currents and the lateral mixing. J. M. Res. 8 (2).

1950 a: Wind-driven currents and surface contours in an enclosed ocean and the lateral mixing. Geophysical Notes (Geophysical Institute, Tokyo University 3. No. 12.)

1950 b: Drift currents in an enclosed ocean Part I. Geophysical Notes (Geophysical Institute, Tokyo University) 3, No. 23.

SAITO, Y., 1951: On the oscillation of water in a canal generated by wind action (Part I). The Journal of the Tokyo University of Fisheries, vol. XXXVIII, No. 1.

Journal of the Tokyo University of Fisheries Vol. 38, No. 2, 1951.

ON THE FITTING, THE SUMMATION, AND AN APPLICATION OF THE LOGISTIC CURVE

Tomokichi YOSHIHARA

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About a hundred years ago VERHULST found the Logistic Curve as a growth curve of human population. In 1920, PEARL and REED[1] rediscovered the same independently. The properties of this curve were investigated by many workers. I shall show in this paper some properties and an application of the curve.

I. A graphic method of determining the parameters of the Logistic type curves.

E. A. RASOR[2] gave a nomograph which enables us to calculate the parameters of L-curve*, M. MASUYAMA[3] remarked a method in a footnote of his paper and gave the name "The Method of Finite Differences Diagram". Some time ago I made an investigation independently and devised the similar method but did not published it. L. A. WALFORD[4] gave a new graphic method of describing the growth of animals. His method is to transform the growth curve into a straight line. By means of similar method we can transform L-curve into a straight line. MASUYAMA and RASOR considered nothing but the case when the slope of line is smaller than unity. If the line passes the origin or does not cross the bisector of coordinates axes in the first quadrant the growth curve will not be represented by L-curve. I found that in these cases we have several types of the Logistic type curves which WILSON and PUFFER[5] classified in their paper. It is the purpose of this section to show briefly the whole approach gained in this previous work.

I–1 The finite difference diagram.

If a set of points (w_1, w_2), (w_2, w_3), $\cdots\cdots(w_{n-1}, w_n)$ arrange on a line, we have the determinante equation

* For the sake of simplicity I shall use an abbreviation L-curve for the Logistic curve.

$$\begin{vmatrix} f(x) & f(x+h) & 1 \\ f(x+h) & f(x+2h) & 1 \\ f(x+2h) & f(x+3h) & 1 \end{vmatrix} = 0, \quad \text{where } w_j = f(x+(j-1)h).$$

It is easy to show that the solution of this equation has the form

$$f(x) = a' + b' e^{-n'x}$$

where a', b', and n' are some constants. By this property we can transform L-curve into a line. Let the equation of L-curve be

$$y = K/(1+\exp(A-Bx)) \quad \cdots\cdots (1)$$

and put

$$z = 1/y. \quad \cdots\cdots (2)$$

When the equation (1) represents L-curve, two constants K and B have to satisfy two conditions

$$0 < K < \infty, \qquad B > 0. \quad \cdots\cdots (3)$$

Given three equidistant values of x, we have three values of y or z, namely z_0, z_1 and z_2. Plot two points $Q(z_0, z_1)$ and $R(z_1, z_2)$, and draw a straight line

$$v = p + qu \quad \cdots\cdots (4)$$

connecting these two points (Fig. 1). Let the bisector of two coordinate axes be $v=u$, and denote by Ω the point of intersection of the line (4) and the bisector, then the coordinates of Ω are $(1/K, 1/K)$.

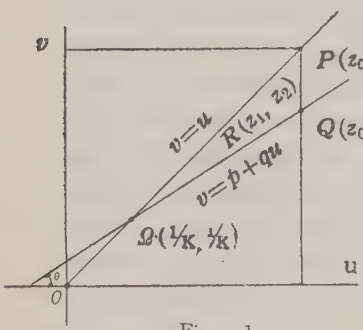

Fig. 1

If there is a set of $(n-1)$ points, we have $(n-1)$ relations

$$z_{i+1} = p + qz_i \qquad (i = 1, 2, \cdots\cdots n-1)$$

and two constants p and q are given, by the mathod of least squares, as follows

$$p = (1-e^{-Bh})/K, \quad q = e^{-Bh} \quad \cdots\cdots (5)$$

hence

$$K = (1-q)/p.$$

Let the angle between the line $v=p+qu$ and abscissa be θ then

$$0 < q < 1 \quad \text{or} \quad 0 < \theta < 45°.$$

WILSON and PUFFER[5] represented L-curve by the equation

$$y = 1/(a+be^{-nx}). \quad \cdots\cdots (6)$$

Put $c=e^{-nx}$, then we have $z=a+bc$. Given three values of z, *e. g.* z_0, z_1 and z_2 at time 0, x and $2x$, we get relations

$$z_0=a+b, \quad z_1=a+bc, \quad z_2=a+bc^2. \quad \cdots\cdots\cdots\cdots (7)$$

As we are dealing with the growth data we can assume $z_0>z_1>z_2$. Put

$$\alpha=z_0-z_1>0, \quad \beta=z_1-z_2>0,$$

$$D=z_0-2z_1+z_2=\alpha-\beta, \quad E=z_0z_2-(z_1)^2$$

then we get from relations (7)

$$a=\frac{E}{D}=z_1-\frac{\alpha\beta}{\alpha-\beta}, \quad b=\frac{\alpha^2}{\alpha-\beta}, \quad c=\frac{\beta}{\alpha} \quad \cdots\cdots\cdots\cdots (8)$$

According to the sign of D and E we can classify the Logistic type curves into five types which WILSON and PUFFER classified. These five types are given in Table 1.

Table 1.

Hypothetical examples of various types of the Logistic type curve and their formulas and names where $\alpha=z_0-z_1$, $\beta=z_1-z_2$, $D=\alpha-\beta$, $a=z_0-b$, $b=\alpha^2/D$, $c=\beta/\alpha$.

Type	y_1	y_2	z_1	z_2	α		D	a	b	c	Formulas	Names of curves
I	1.5	2	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$y=\frac{3}{1+2^{1-x}}$	Hypermalthusian with Saturation
II	1.5	$\frac{9}{4}$	$\frac{2}{3}$	$\frac{4}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{9}$	0	1	$\frac{2}{3}$	$y=1.5^x$	Malthusian no Saturation
III	1.5	2.4	$\frac{2}{3}$	$\frac{5}{12}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{-1}{3}$	$\frac{4}{3}$	$\frac{3}{4}$	$y=\frac{3}{-1+4(3/4)^x}$	Hypermalthusian no Saturation
IV	1.5	3	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$-\infty$	$+\infty$	1	$y=\frac{1}{1-x/3}$	Critical no Saturation
V	1.5	9	$\frac{2}{3}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{5}{9}$	$\frac{-2}{9}$	$\frac{3}{2}$	$\frac{-1}{2}$	$\frac{5}{3}$	$y=\frac{2}{3-(5/3)^x}$	Hypercritical no Saturation

1-2 Graphic determination of parameters.

Take a point $P(z_0, z_0)$ in the first quadrant and join two points $O(o, o)$ and P. (Fig. 2). If the point Ω lies on the segment OP, the data can be represented by L-curve, but in other case it will be represented by other curves. In Table 1, the first example is L-curve. In the second example, the point Ω consists with the origin, thus data can be represented by the exponential curve. In the third example, the point Ω appears in the third quadrant and

the data can be represented by a Logistic like curve which has an infinite value for the finite value of x. In the fourth example, the line $v=p+qu$ is parallel to the line OP, in this case the data can be represented by a hyperbola. Finally in the last example the point lies on a segment which elongated the line OP, and the data can be represented by the Logistic like curve which has an infinite value for finite x.

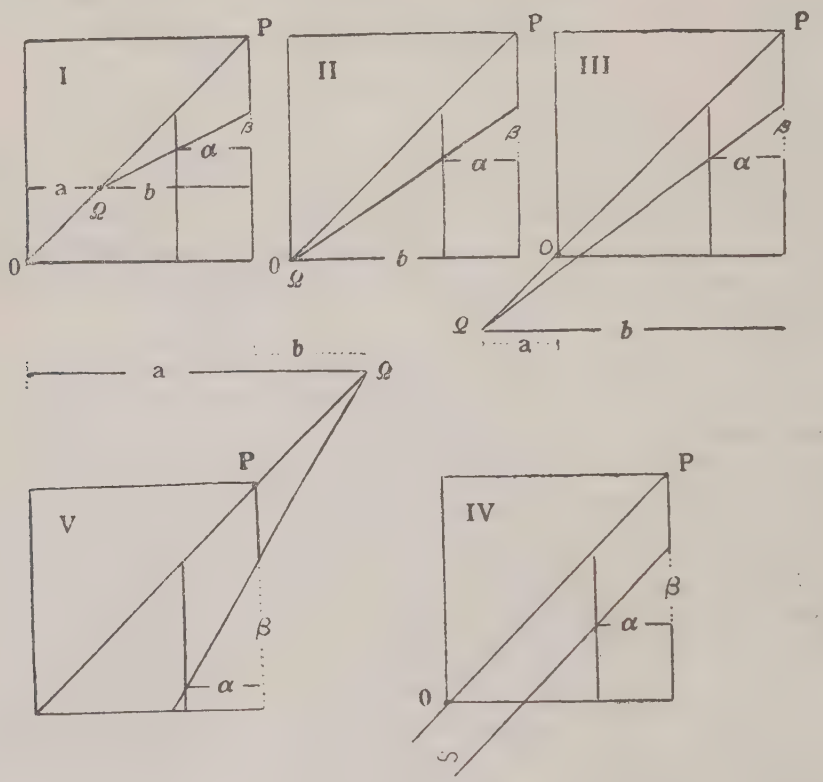

Fig. 2

As stated before the point Ω has the coordinates $(1/K, 1/K)$. From equation (7) we obtain $b=z_0-a$, from equation (8) $c=\beta/\alpha$, and from equations (1) and (6) $1/K=a$, thus three parameters a, b and c can be determined easily. In the following it will be shown that the values of these parameters can be read from the finite difference diagram.

As an application of the finite difference diagram the parameters of L-curve for the growth of population of the United States of America are determined. The data are given in Table 2.

Denote by t the calender years, by t_0 the origin of time, by h the time

interval and put

$$x=(t-t_0)/h,\ t_1=t_0+h,\ t_2=t_1+h,$$

then equation (6) becomes

$$z=1/y=a+be^{-nx}=\left(z_1-\frac{\alpha\beta}{\alpha-\beta}\right)+\frac{\alpha\beta}{\alpha-\beta}exp\{-(t-t_1)/h\} \quad \cdots\cdots\cdots (9)$$

where z_0, z_1 and z_2 correspond to t_0, t_1 and t_2.

Take $h=10$ years and plot the values of z from $t=1790$ to $t=1940$. (Fig. 3). Then we have a translated line but the angle θ of this line is about 45°, and the point Ω can not be determined accurately. If we note the relation $\tan\theta=q=e^{-Bh}$ we know that when we take larger value of h, the angle θ decreases. Taking $h=50$, $t_0=1830$, $t_1=1880$ and $t_2=1930$ we obtain another translated line as shown in Fig. 3. From this diagram we can obtain the values

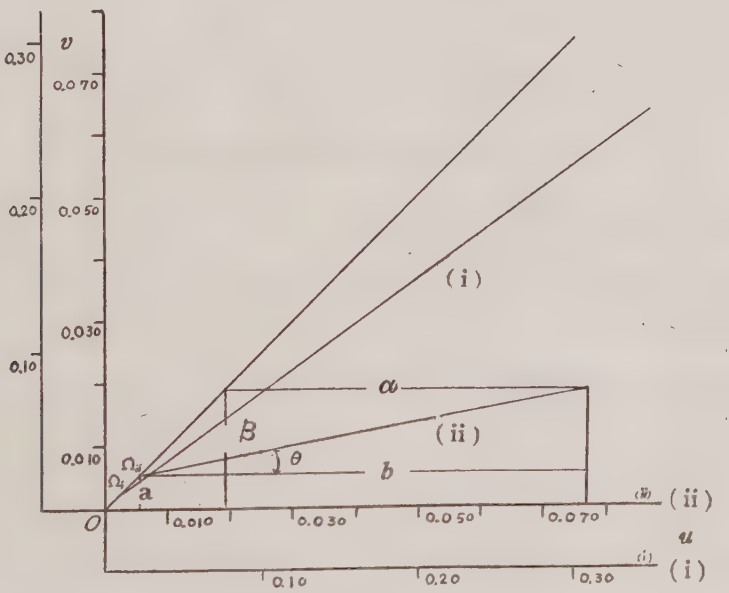

Fig. 3

$a=0.0055$, $b=0.0722$, $c=0.192$ and $\alpha\beta/(\alpha-\beta)=z_1-a=0.0144$.
Thus from equation (9)

$$y=(0.0055+(0.0144)(0.192)^{(t-1880)/50})^{-1}$$
$$=182(1+exp(3.928-0.330(t-1790)/10))^{-1} \quad \cdots\cdots\cdots (10)$$

Table 2.

The growth of population of the United States since 1790.

t	x	$y(\times 10^6)$	$z=1/y$	Calculated values	
				$Eq.(10)$	$Eq.(11)$
1790	0	3.9292	0.25582	3.51	3.785
1800	1	5.3084	18939		
10	2	7.2399	13885		
20	3	9.6385	10431		
30	4	12.8660	07770	12.45	12.857
40	5	17.0695	05858		
50	6	23.1919	04312		
60	7	31.4433	03188		
70	8	38.5584	02573		
80	9	50.1558	01994	50.4	50.089
90	10	62.9477	01589		
1900	11	75.9946	01316		
10	12	91.9723	01083		
20	13	105.7106	00943		
30	14	122.7750	00812	121.2	122.94
40	15	131.6693	00760		

The exact solution which is obtained from equation (8) is

$$y=196.85\,(1+exp\,(3.932-0.3174\,(t-1790)/10))^{-1} \quad \cdots\cdots\cdots \quad (11)$$

Calculated values of y by each equation are shown for comparison in Table 2.

2. On the summation of the Logistic curves and the mean constant Logistic curve.

In 1927, PEARL and REED[6] had treated a problem on the summation of L-curve and in 1931, MERREL[7] got the mean constant Logistic curve in the study of similar problem. I have solved, in general, the problem of the summation of growth curve and the mean constant curve.

Let the function $F(x)$ be a growth curve of a biomass (e.g. number of individuals of an animal population or the weight of individuals of animal). In ordinary circumstance, $F(x)$ is a monotonously increasing function of x but does not increase infinitely and has a maximum value L, say. If the function $F(x)$ is differentiable, denote the derivative of it by $f(x)$, then we have the relation $F(x)=\int_{-\infty}^{x} f(t)\,dt$. Without loss of generality we can assume $F(-\infty)=0$ and the function $f(x)$ has the total frequency L if we consider it as a frequency function.

If we define a function $G(x)$ by the relation $G(x)=F(x)/L$, we can treat $G(x)$ as a distribution function. And by the theory of sampling we can find the distribution function of random variable

$$Z=c_1X_1+\cdots\cdots\cdots\cdots+c_nX_n,$$

where we assume that each random variable X_j has a distribution function $G_j(x)$ and independent each other and c_j is some constant.

It is known that the characteristic function of linear function of n independent random variables is the product of their $c.f.$ and if random variable X_j has a distribution function $G_j(x)$, random variable c_jX_j have a distribution function $G_j(x_j/c_j)$ and $c.f.$ of Z is given by $M(t)=\prod_{j=1}^{n} M_j(c_jt)$ where $M_j(t)$ is a $c.f.$ of $G_j(x)$.

For the summation and the mean of growth curve we shall use above properties. Assume that there are n populations whose growth curve are $F_j(x)$ and their maximum value are $L_j(n=1,2,\cdots\cdots n)$, then $G_j(x)=F_j(x)/L_j$ is a distribution function. As the sum of growth curves we shall use the distribution function of $Z=\sum_{j=1}^{n} c_jX_j$ where $c_j=L_j/\sum_{j=1}^{n} L_j$. Recently I knew that Sittig (1948) found the superposition of two frequency distributions but I could not referred his paper.

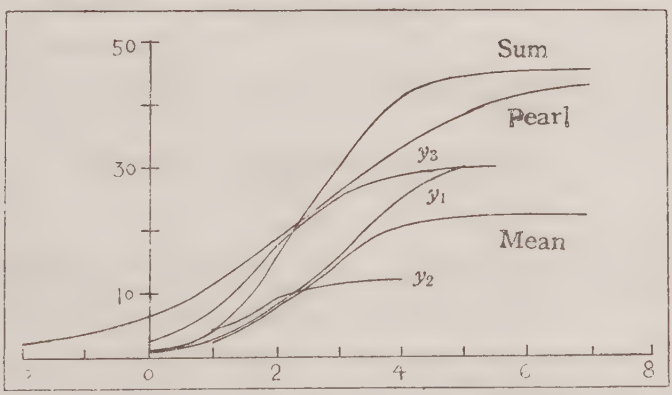

Fig. 4

We shall apply this method to the example which is given by Wilson and Luyter (1925). The data are as follows:

time	x	0, 1, 2
first population	y_1	1, 3, 8
second population	y_2	1, 4, 9.

From these two populations make a composed population $y_3=y_1+y_2$, *i. e.*

third populations y_3 2, 7, 17.

The values of parameters for these populations are

$L_1=33.000$	$L_2=88/7=12.571$	$L_3=91/3=30.333$
$a_1=3.466$	$a_2=2.449$	$a_3=2.668$
$b_1=1.163$	$b_2=1.686$	$b_3=1.447.$

We should notice that the total of the first and the second population at time $x=0, 1$ and 2 equal to the value of third population at the same time but the sum of maximum value of the former *i. e.* $L_1+L_2=45.571$ is different from the maximum value of the later *i. e.* $L_3=30.333$. (Fig. 4).

In order to find a growth curve which has a maximum value $L=L_1+L_2$ we shall find the superposition of two curves.

If there are n populations whose d. f. is

$$G_j(x)=\frac{F_j(x)}{L_j}=\frac{1}{1+exp(a_j-b_jx)} \qquad (j=1, \cdots\cdots, n)$$

then $$g_j(x)=G_j'(x)=\frac{b_jexp(a_j-b_jx)}{(1+exp(a_j-b_jx))^2} \qquad (j=1, 2, \cdots\cdots n)$$

To find the *c.f.* of random variable c_jX_j we have to calculate the integral

$$\int_{-\infty}^{\infty}\frac{b_j}{c_j}\cdot\frac{exp(a_j-b_jx/c_j)}{(1+exp(a_j-b_jx/c_j))^2}\cdot e^{it\cdot x}dx$$

and *c.f.* $M(t)$ of Z is the product of these integrals. To perform this integral put $1+e^{a-bx}=z$, then

$$M(t)=exp\sum_{j=1}^{n}\frac{ita_jc_j}{b_j}\prod_{j=1}^{n}\Gamma\left(1+it\frac{c_j}{b_j}\right)\cdot\Gamma\left(1-it\frac{c_j}{b_j}\right)$$

$$=e^{it\Delta}\prod_{j=1}^{n}\left(\frac{c_j}{b_j}\pi t/\sinh\frac{c_j}{b_j}\pi t\right) \qquad \cdots\cdots (12)$$

where

$$\Delta=\sum_{j=1}^{n}a_jc_j/b_j$$

In order to get the exact solution of composed growth curve we have to apply the inversion formula to $M(t)$, but the result is not L-curve, and it has not a convenient form for numerical calculation. Therefore an approximate solution was obtained.

For this purpose the cumulants γ_j and moments m_j of the distribution function of Z are given by taking the logarithum of $M(t)$. Taking the logarithum of both sides of equation (12), we have

$$\log M(t) = it\Delta + \sum \log \Gamma(1+itc_j/b_j)\,\Gamma(1-itc_j/b_i)$$

$$= it\Delta + S_2 R_2 (it)^2 + \tfrac{1}{2} S_4 R_4 (it)^4 + \tfrac{1}{3} S_6 R_6 (it)^6 \cdots\cdots\cdots\cdots \quad (13)$$

where $$R_k = \sum_{j=1}^{n} (c_j/b_j)^k, \quad S_k = \sum_{j=1}^{\infty} j^{-k}$$

Hence ([8], p. 60-63).

$$\gamma_1 = \gamma_3 = \gamma_5 = \cdots\cdots\cdots\cdots = 0 \qquad m_1 = m_3 = m_5 = \cdots\cdots\cdots\cdots = 0.$$

$$\gamma_2 = 2!\,S_2 R_2 = \pi^2 R_2/3, \qquad m_2 = \pi^2 R_2/3$$

$$\gamma_4 = 4!\,S_4 R_4/2 = 2\pi^4 R_4/15, \qquad m_4 = \pi^4 (5R_2^2 + 2R_4)/15 \qquad (*)$$

$$\gamma_6 = 6!\,S_6 R_6/3 = 16\pi^6 R_6/63, \qquad m_6 = \pi^6 (16R_6 + 35R_2^3 + 42R_2 R_4)/63$$

Equation (13) may be rewritten;

$$M(t) = \exp(it\Delta - S_2 R_2 t^2) \cdot \exp\left[S_4 R_4 (it)^4/2 + S_6 R_6 (it)^6/3 + S_8 R_8 (it)^8/4 \cdots\cdots\right]$$

$$= \exp(it\Delta - S_2 R_2 t^2) \cdot \left[1 + S_4 R_4 (it)^4/2 + S_6 R_6 (it)^6/3 + (2S_8 R_8 + S_4^2 R_4^2)(it)^8/8 + \cdots\cdots\right]$$

---

* We can find the formulae of the k-th moment about mean ($m = a/b$) of the distribution function $G(x) = (1+e^{a-bx})^{-1}$ as follows: By definition

$$m_k = \int_{-\infty}^{\infty} (x-m)^k dG(x) = \int_{-\infty}^{\infty} (x-m)^k b e^{a-bx} (1+e^{a-bx})^{-2} dx$$

$$= \int_{-\infty}^{\infty} \left(\frac{a+y}{b} - m\right)^k \frac{e^{-y}}{(1+e^{-y})^2} dy = \frac{1}{b^k} \int_{-\infty}^{\infty} \frac{y^k e^{-y}}{(1+e^{-y})^2} dy$$

hence, $m_k = 0$ for odd k, and for even $k (= 2p$, say)

$$m_{2p} = \frac{2}{b^{2p}} \int_0^{\infty} \frac{y^{2p} e^{-y}}{(1+e^{-y})^2} dy = \frac{2(2p)!}{b^{2p}} \sum_{n=0}^{\infty} (-1)^n \binom{n+1}{n} (n+1)^{-2p-1}$$

$$= (2^{2p} - 2) b^{-2p} B_p \pi^{2p}$$

where B_p is BERNOULLI's number and its values are

$$B_1 = 1/6, \quad B_2 = 1/30 \quad B_3 = 1/42, \quad B_4 = 1/30, \cdots\cdots.$$

and the frequency function of Z is given by

$$h(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}M(t)\,e^{-itx}dx$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-it(x-\Delta)-B_1\pi^2R_2t^2}\cdot\left[1+\frac{1}{2}S_4R_4t^4+\frac{1}{3}S_6R_6(it)^6+\cdots\cdots\right]dt\cdots\cdots(14)$$

where $B_1=1/6$ and $S_2=\pi^2/6$, $S_4=\pi^4/90$, $S_6=\pi^6/945$ $S_8=\pi^8/9450$. etc.

Next, in order to transform the equation (14) to a form convenient for the numerical calculation, we shall use the fact that a normal distribution of mean Δ and variance $\sigma^2=m_2$ can be be represented by

$$\Phi(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-t(x-\Delta)-B_1\pi^2R_2t^2}dt=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\Delta)^2/2\sigma^2}\quad\cdots\cdots\cdots\cdots(15)$$

hence differentiating $2j$ times we have

$$\Phi^{(2j)}(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}(it)^{2j}\,exp\{-it(x-\Delta)-B_1\pi^2R_2t^2\}\quad\cdots\cdots\cdots\cdots(16)$$

By means of (15) and (16) we have from equation (14)

$$h(x)=\Phi(x)+\frac{1}{2}S_2R_2\Phi^{(4)}(x)+\frac{1}{3}S_6R_6\Phi^{(6)}(x)+\frac{1}{8}(2S_8R_8+S_4^2R_4^2)\Phi^{(8)}(x)+\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(17)$$

Equation (17) can be transformed into a simple form if we use the normalised form.

Put $$\Phi_0(t)=\frac{1}{\sqrt{2\pi}}e^{-t^2/2},\quad\text{where}\quad t=\frac{x-\Delta}{\sigma}$$

then $$\Phi(x)=\Phi_0(t)/\sigma,\quad\Phi^{(2j)}(x)=\Phi_0^{(2j)}(t)/\sigma^{2j+1}$$

Table 3. Constants of the sum curve.

$c_1=0.7242$	$c_2=0.2758$	$\Delta=2.559$
$R_2=0.4145$	$R_4=0.1511$	$R_6=0.0583$
$\sigma=1.168$	$R_4/20R_2^2=0.0440$	$R_6/105R_2^3=0.00777$

Table 4. Calculation of the sum curve.

time	component curve		sum curve						
x	y_1	y_2	y_3	t $=0.856x-2.191$	integral term	$0.0440\Phi'''_0$	H(x)	(L_1+L_2) H(x)	Pearl's method
−2				−3.903	0.0000	−0.0005	−0.0005	—	1.80
−1				−3.047	12	32	44	0.2	3.30
0	1	1	2	−2.191	143	63	206	1.0	6.49
1	3	4	7	−1.335	910	− 119	791	3.6	11.44
2	8	9	17	−0.489	3157	208	3365	15.3	18.37
3	15.6	11.7	25.5	0.377	6480	174	6654	30.3	26.23
4	25.2	12.1	29.0	1.234	8906	151	9057	41.2	33.41
5	30.1			2.090	9817	− 57	9760	44.4	38.32
6				2.946	9984	− 38	9946	45.3	41.85
7				3.802	9999	− 6	9993	45.5	43.44
∞	33.0	88/7	91/3					45.571	45.571

The mean curve is obtained as $(L_1+L_2)\,H(x)/2$.

hence equation (17) becomes

$$h(x)=\frac{1}{\sigma}\left[\Phi_0(t)+\frac{1}{20}\cdot\frac{R_4}{R_2^2}\Phi_0^{(4)}(t)+\frac{1}{105}\cdot\frac{R_6}{R_2^3}\Phi_0^{(6)}(t)+\cdots\cdots\right] \quad \cdots\cdots\cdots (18)$$

where $\sigma^2=\pi^2R_2/3$

Integrating the equation (18) from $-\infty$ to x, we have the distribution function of Z namely

$$H(x)=\int_{-\infty}^{x}h(x)\,dx$$
$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t}e^{-t^2/2}dt+\frac{1}{20}\cdot\frac{R_4}{R_2^2}\Phi_0'''(t)+\frac{1}{105}\cdot\frac{R_6}{R_2^3}\Phi_0^{(5)}(t)+\cdots\cdots$$
$$\cdots\cdots\cdots (19)$$

This is the result which we want.

In the following we shall find the sum and the mean curve for an example given previously. The calculated values for various constants are shown in Table 3 and the values of y's in Table 4. For comparison I have calculated the constants of the sum curve by Pearl's method which are given by the relations;

$$L=L_1+L_2,\quad \frac{aL}{b}=\frac{a_1L_1}{b_1}+\frac{a_2L_2}{b_2},$$

$$\frac{L}{b^2}=\frac{L_1}{b_1^2}+\frac{L_2}{b_2^2}+3\left(\frac{a}{b}-\frac{a_1}{b_1}\right)^2L_1+3\left(\frac{a}{b}-\frac{a_2}{b_2}\right)^2L_2$$

Numerical values of constants are $a=1.795$ $b=0.7017$ $L=45.571$.

3. On BARANOV's paper

About thirty years ago BARANOV[11] had developed a theory on fish population. Recently his original paper was translated into English by Prof. W. E. RICKER. BARANOV assumed that there is a linear relation between the length and age of fish. I have changed this fundamental assumption by the other one and got some results. I wish to return herewith my hearty thanks to Professor W. E. RICKER for his kindness which gave me BARANOV's paper.

Denoting by the letter n the abundance of any group of fish and by the letter t the age of fish, BARANOV assumed that the decrease dn in the number of fish in small time dt is proportional to the abundance of that group and put $dn/dt=-k_1n$, where k_1 is a constant. Furthermore he assumed that the increase of length of fish is proportional to t and put $t=rl$, where r is a constant. From these two equations, after intergrating, we have the relation between l and n, namely $n=n_0e^{-kl}$, where the constant $k=k_1r$ is the coefficient of decrease and n_0 is the initial numbers of individuals of population. k can be determined from the formula $k=(log\ n_1-log\ n_2)/(l_1-l_2)$ where we assume that there is two groups of fish at different time and the first group contains of n_1 fish of length l_1 and the second group consists of n_2 fish of length l_2. Having determined the magnitude of n_1 and n_2 he defined the annual decrease of fish. It is given by the expression $\varphi=(n_1-n_2)/n_1$.

There is a lower limit of body length, and fish whose length is less than this minimum value do not enter into the composition of the commercial catch. This length L is called the minimum length. Denoting by R the whole number of fish of commercial size that is fish whose length is greater than L and by P the whole weight of fish population of commercial size, he reduced a relation, assuming that the body weight of fish is proportional to the cube of their length, $P=const.\ RL^3q$ (*), where q is some function of kL. It is easy to see that the product const. L^3q is the mean weight of fish. It may be permissible to use formula (*) to obtain the coefficient k, if the weight of fish of minimum commercial size and the weight of fish in the catch be known. He calculated the values of k, especially for the value of which may apply to the upper limit of mortality of the fish at the time when the industry was only feebly developed and he had analysed the case of the joint action of natural death and fishing.

In Japan, Spring Herrings are caught in the Hokkaido District from ages 3 to 8 years. The origin of this fishery is very old and statistics of catch in weight is obtained since 1870. In 1910, Hokkaido Fishery Experiment Station began the investigations of this fishery and made the statistics

of age composition which was calculated from scales. The growth of herring of each age was determined by KAWAI using the scale reading of 8 age herrings, TSUDA published a table of the mean weight of each age. From these data I have prepared Table 5 from which we know that the increase of length of each age is not constant and the growth curve will be fitted more suitably by L-curve. Therefore I assumed that the growth of fish length can be represented by L-curve and using it instead of the linear relation similar results were obtained. I shall show in the following a generalisation of BARANOV's theory when we use L-curve.

Table 5.

Vital statistics of herring

Age(x)		2	3	4	5	6	7	8
Length(l) (cm.)	obs.	20	24	27	28.3	30	31	32.2
	cal.	19.4	24.8	27.4	29.2	30.6	31.6	32.2
Age composition (%)		–	7.5	15.7	32.4	21.5	12.9	10.1
Mean body weight (Monmme)		–	41.0	65.5	74.9	85.5	95.2	108.1

3-1. Relations between the number of individuals and length of fish when the length of fish can be represented by the Logistic equation.

Let us assume that the length of fish of an age x at a time t can be represented by L-curve

$$l=\frac{M}{1+exp(a-bt)} \qquad \cdots\cdots(20)$$

where M, a, b are constant and their meaning are given already.

Integrating the equation $dn/dt=-k_1 n$ we have

$$n=n_0 e^{-k_1 t} \qquad \cdots\cdots(21)$$

from equations (20) and (21) we get

$$n=n_0\left(\frac{M}{l}-1\right)^{\frac{k_1}{b}} e^{-ak_1}$$

where $k=k_1/b$. Using the data of Table 6 we have

$$M=33.33, \quad a=0.276 \quad \text{and} \quad b=0.448$$

As the numbers of fish are decreasing exponentially above the age 5, we have $k_1 \fallingdotseq 0.5$, hence

$$n=n_0\left(\frac{M}{l}-1\right)^{1.12}e^{-0.307} \qquad \cdots\cdots(22)$$

The annual decrease of fish population is given, by the definition of φ,

$$1-\varphi=\alpha^k \quad \text{where} \quad \alpha=(M-l_2)/(M-l_1)$$

Since the body length of an age 3 fish is 24 cm. and that of an age 4 years is 27 cm., we have $(1-\varphi)^{1/k}=(33.33-27)/(33.33-24)=0.675$ hence $\varphi=0.355$.

The total number R of fish of commercial size is given by

$$R=n_0e^{-ak}\int_L^M\left(\frac{M}{l}-1\right)^k dl$$

and the whole weight P is given by

$$P=const\cdot n_0e^{-ak}\int_L^M\left(\frac{M}{l}-1\right)^k l^3dl$$

If we assume that $l=Mz$ and $L=M\beta$ then $0<\beta<1$, and the mean weight is given by

$$W=P/R=\text{Const. } L^3\,q_k$$

where

$$q_k=\int_\beta^1\left(\frac{1}{z}-1\right)^k z^3dz\Big/\beta^3\int_\beta^1\left(\frac{1}{z}-1\right)^k dz$$

For the integral value of k, q is given as follows:

$$k=1, \quad q=(1-4\beta^3+3\beta^4)/12(\beta-1-\log\beta)\beta^3$$

$$k=2; \quad q=(1-6\beta^2+8\beta^3-3\beta^4)/(1-\beta^2+2\beta\log\beta)\beta^2$$

$$k=3; \quad q=(1-\beta+6\beta^2-4\beta^3+\beta^4)/2(2\beta^3+3\beta^2-6\beta+1-6\beta^2\log\beta)\beta.$$

For non-integral values of k, simple formula of q can not be obtained.

For the herring the mean weight is 79.09 monmme (1 monmme= 3.75 gr). If we take 45 monmme for 3 age or 60 monmme for 4 age as minimum body weight corresponding to L, then $q=1.76\sim1.32$. On the other hand $\beta=(25\sim27)/33.33=0.75\sim0.81$. Using these value, the value of k is approximately 1.1 and this is nearly equal to the value above given. (See eq. (22)).

The comparison of present condition of fishery and that of remote times can not be obtained since there is no correct datum concerning the vital statistics before 1910.

LITERATURE

1. Pearl, R. and L. J. Reed. On the rate of growth of the population of the United States since 1790 and its mathematical representation. Proc. Nat. Acad. Sci. 6, 1920, 275–288.
2. Rasor, E. A. The fitting of logistic curve by means of a nomograph. J. Amer. Stat. Assoc. 44 (248), 1949, 548–553.
3. Masuyama, M. A graphical method of estimation of parameters contained in the Langevin's non-stochastic equation. Staudy of Math. Stat. (Tokei Suri Kenkyu, in Japanese) 2 (2), 1948, 28–29.
4. Walford, L. A. A new graphic method of describing the growth of animals. Biol. Bull. 90 (2), 1946, 141–147.
5. Wilson, E. B. and R. R. Puffer. Least squares and law of population growth. Proc. Nat. Acad. Sci. Art and Sci. 68, 1932, 285–382.
6. Pearl, R. and L. J. Reed. On the summation of logistic curve. J. Roy. Stat. Soc. 1927, 729–746.
7. Merrel, M The relationship of individual growth to average growth. Human Biology 3, 1931, 37–70.
8. Kendall, M. G. The advanced theory of statistics, vol. I. 1948.
9. Wilson and Luyter. The population of New York City and its environs. Proc. Nat. Acad. Sci. 11, 1925, 137–143.
10. Kostitzin, V. A. (Translated by T. H. Savory) Mathematical Biology 1939, pp. 65–66.
11. Baranov, T. I. (Translated by W. E. Ricker, Typewritten) On the question of biological basis of fisheries. pp. 53. Nauchnyi issledovatelskii iktiologisheskii Institute, Izvestiia, 1 (1), 1918, 81–128. (Moscow).

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DEHYDRATION OF THE AGAR-AGAR GEL BY ADSORPTION.

PART 1. DEHYDRATION BY THE "BENTONITE".

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On the manufacture of the agar-agar, the dehydration of the agar-agar gel is the most important process. The methods available at the present time for the dehydration of it are as follows:

1. Natural freezing and drying under the freezing weather.
2. Refrigeration.
3. Hydric pressure.
4. Spray drying.
5. Cataphoresis.

Except the natural freezing and drying method, every method has its merit, all the others need many mechanical processes.

Then the authors tried to dehydrate the agar-agar gel by the adsorption of the "Bentonite" or such adsorbable substances as Fuller's earth. Though such a method is primitive, it is very simple and, what is better, the manufactured agar-agar is excellent, not expensive, and not limited by the weather and the region, and the mechanical processes is not needed.

EXPERIMENTAL

The agar-agar gel was prepared by the ordinary method from the seaweeds (*Gelidium Amansii* LAMS). This agar-agar gel (contains 2% agar-agar) was sliced into a size of 10 cm. square and 0.5 cm. in thickness. As Fig. 1. shows the sliced agar-agar gel was wrapped in a cloth, and placed between the two layers of 100 g. of "Bentonite" that was uniformly formed in a wooden frame, and a weight of 1 kg. was placed on a lid of a proper size for the frame. In this experiment the air-dried "Bentonite" (with 11.6% moisture) was used.

In this case the agar-agar gel dehydrated by the adsorption of "Bentonite" was weighed each hour, and the percentage of dehydration was calculated. The results were shown on the table 1.

As is seen on the table 1, it was dehydrated after one hour 40% more, after 6 hours 80%, and after 20 hours 90% more.

In this case if the moistened "Bentonite" was exchanged for dried "Bentonite" each hour, after the third exchange, it was dehydrated 95%. The results were shown on the table 2.

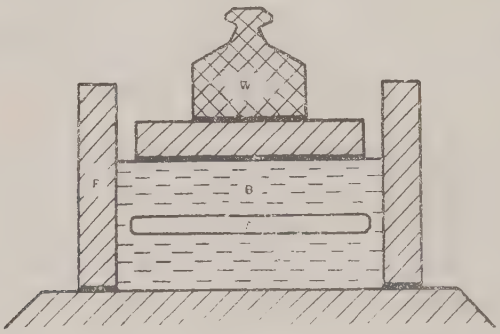

Fig. 1. Diagram showing the dehydration aparatus used in the present investigation. A, Agar-agar gel; B, Bentonite; F, Frame; W, Weight.

Table 1. The dehydrated percentage of the agar-agar gel.

Hour	1	2	4	6	8	10	12	14	16	18	20	22	24
Dehydrated percentage	42	60	75	80	84	87	88	89	90	91	91	92	92

Table 2. The dehydrated percentage ("Bentonite" was exchanged each hour).

A number of times of the exchange of "Bentonite"	I	II	III
Dehydrated percentage	42	80	95

As is seen on the table 2, when the moistened "Bentonite" was exchanged three times, the purpose of the dehydration was satisfactorily achieved in a short time.

The moistened "Bentonite" can be repeatedly used through drying it in the sun or other drying method, and as the agar-agar gel and the "Bentonite" can be piled high one on another, they will not occupy much floor space.

THE INGREDIENTS OF THE AGAR-AGAR BY THIS METHOD

The ingredients of the agar-agar by this adsorption method was compared with those of the agar-agar dried in the air, and the results were shown on the table 3.

Table 3. The ingredients of the agar-agar.

Dehydration method	Moisture %	Ash %	Total nitrogen %	Jelly strength g/cm²
Agar-agar dried in the air	21.3	7.0	1.23	——
Agar-agar dehydrated by this method	22.3	2.7	0.30	650

As is seen on the table 3, the contents of the ash and the total nitrogen of the agar-agar prepared by the adsorption method showed a decrease as compared with the agar-agar dried in the air as it was.

The strength of the jelly of the agar-agar prepared by this adsorption method was very high.

The impurities in the agar-agar gel were removed with the water adsorbed.

Then the agar-agar gel could be purified simultaneously with dehydration.

DEHYDRATION RATIO OF THE AGAR-AGAR GEL ON THIS EXPERIMENT

Table 1 was graphically showed on Fig. 2.

If the dehydration ratio of the agar-agar gel shown on Fig. 2 was applied in the same manner as the case of the diffusion of heat, the water existed in the center of the agar-agar gel after thours was given as follows:-

$$u = \frac{2c}{\sqrt{\pi}} \int_0^{\frac{d}{a\sqrt{t}}} exp\{-B^2\}\, dB \qquad (1)$$

$u =$ water content

c = initial water existed in the agar-agar gel.
d = the half thickness of the agar-agar gel.
a = constant.

The above formula was renewed as follows:-

$$\frac{u}{c}=\frac{2}{\sqrt{2\pi}}\int_0^{\frac{\sqrt{2}d}{a\sqrt{t}}} exp\{-\frac{t^2}{2}\}\,dt \qquad \cdots\cdots(2)$$

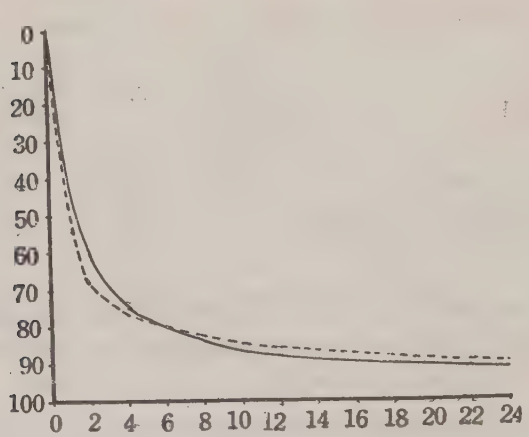

Fig. 2. Dehydration ratio. Ordinate, Dehydration percentage; Abscissa, hour; Solid line, actual value; brocken line, caliulated one.

Table 4. Dehydration ratio from the empirical formula.

t	$\frac{c-u}{c}$	$\frac{0.6}{\sqrt{t}}$	$\frac{1}{\sqrt{2\pi}}\int_0^{\frac{0.6}{\sqrt{t}}} exp\{-\frac{t^2}{2}\}\,dt$	$1-\frac{2}{\sqrt{2\pi}}\int_0^{\frac{0.6}{\sqrt{t}}} exp\{-t^2\}dt$
0	0	∞	0.5	0
2	0.60	0.43	0.166	0.67
4	0.75	0.30	0.118	0.77
6	0.85	0.245	0.097	0.80
8	0.84	0.212	0.083	0.83
10	0.87	0.190	0.075	0.85
12	0.88	0.174	0.069	0.86
14	0.89	0.160	0.064	0.87
16	0.90	0.150	0.059	0.88
18	0.905	0.141	0.056	0.89
20	0.91	0.131	0.053	0.89
22	0.92	0.128	0.051	0.90
24	0.92	0.122	0.049	0.90
(i)	(ii)	(iii)	(iv)	(v)

The dehydration ratio $\frac{c-u}{c}$ was approximately given from the graph of Fig. 2 and on the column (ii) of the table 4.

If the formula (2) was integrated, it was given on the column (iv) of the table 4. The calculated value of the dehydration ratio was obtained from this. It was shown on the column (v) of the table 4 and showed with the broken line on Fig. 2. It agreed with the empirical value.

Therefore, the formula (1) might be used as the empirical formula.

Numerecal value of $\frac{\sqrt{2}}{a} d$ was 0.6 which was obtained from the formula (2) using the value of the column (ii), and this value was applied to the calculation of column (iii).

Résumè.

(1) In this experiment, the agar-agar gel was dehydrated after one hour 40% more, after 6 hours 80%, and after 20 hours 90% more by this adsorption of "Bentonite".

(2) When the moistened "Bentonite" was exchanged for dried "Bentonite" each hour, after third exchange it was dehydrated 95%. By this method the dehydration was satisfactorily achieved in a short time.

(3) The agar-agar prepared by this method was very excellent, the contents of the ash and the total nitrogen was very low, and the jelly strength was very high.

Then the agar-agar gel could be purified simultaneously with dehydration.

(4) The moistened "Bentonite" was repeatedly used after dried in the sun or other drying method.

(5) As the agar-agar gels and the layers of "Bentonite" could be piled high one on another, they did not occupy much floor space.

(6) This adsorption method was economical as compared with the other methods e. g. refrigeration, hydric pressure, spray-drying, and cataphresis to which many mechanical processes are necessary.

(7) In this method Fuller's earth or sand could be used.

(8) As the cold district and freezing weather were not necessary to this method, it could be applied everywhere through the country.

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STUDIES ON KAMABOKO-FORMING ABILITY OF MYOSIN

(Part I)

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I. PREFACE

There may be no need for dwelling upon the fact that an important part of food source in this country is dependent upon marine products, especially upon fish. However, catch of fish is not always stationary. Moreover, fish is associated with remarkable tendency to be degradated soon after the catch. The problem of fish preservation, therefore, has become a most serious matter. Although development of freezer today is contributing greatly to solution of this problem, it may not always be suitable for steady supply of sufficient amount of such fish that is cheap enough as to fit to daily consumption of country people, not only because the freezer equipment requires not a small investment but also because frozen fish ought to be distributed as frozen in oder to retain the freshness. On the other hand, in the case with such cheap fish, stabilization by salting or drying is being practised currently at a small expense without any special equipment. However, in respect of palatability, the products by such processes are not so welcomed by consumer on account that properties of the fish have become very different from those of fresh one, especially in the case with fatty fish which readily becomes rancid.

The late Dr. S. TAMARU (1928) who had early esteemed utilization of fish for improvement of the national nourishment, attempted to prepare the so-called "powder of raw fish," *i.e.*, such a special sort of fish-powder that might be always convertible readily into knead products such as, for instance, *Chikuwa*, *Hampen*, or *Kamaboko* by kneading it with water. He has proposed the attempt with reason: on the basis of the statistical data, among processed foods of marine product, knead products, *i.e*, the so-called *Renseihin*, such as *Chikuwa* or *Kamaboko* are most generally welcomed by our populace, since these products are not only cheap but also come to their liking. Indeed, it would not be too much to say that the problem of fish preservation can be almost solved by discovery of any successful method to preserve the knead products whether directly or indirectly.

The procedure proposed by Dr. TAMARU was as follows:

Fresh raw fish is first crushed thoroughly by a disintegrator and diluted with water into a milky suspension. Then is sprayed the suspension in a current of hot air which is finally driven away through bag-filters leaving the fish-powder behind.

It was expected by Dr. TAMARU that, in the case with such spraying, evaporation of water might be performed so quickly that the sprayed material was always kept cold enough by vaporization heat of water, and thus fish might be dried into powder without the protein being subjected to coagulation by heat. In the case of fatty fish, the oil was intended to be removed from the powder subsequently by extraction with a solvent.

However, the present author, being then in charge of the examination of the plan under Dr. TAMARU's leadership, met with such dejecting facts as follows:

(1) It was quite impossible to crush whole raw fish by popular sorts of disintegrator into a state capable to produce an aqueous suspension such as to be sprayed entirely as it is.

(2) Even though a sifted portion from an aqueous suspension of crushed fish could be sprayed into such powder as able to be set into a *Kamaboko*-like mass for a short period immediately after the preparation, the powder lost invariably the ability before long unless it was kept without moisture (HIRANO, 1942 a).

(3) The *Kamaboko*-like mass formed with the powder was recognized as a somewhat different substance from typical *Kamaboko* not only in the point that it could be formed only at a higher protein content but also in the point that it was remarkably poorer in elasticity compared with typical *Kamaboko*.

Although what had led to such results remained unexplained, it was experienced by the author, on the other hand, the fact that raw fish was sometimes dried without losing *Kamaboko*-forming ability when it was desiccated rapidly at low temperature, for instance, under reduced pressure over a concentrated sulfuric acid, as described in a previous paper (HIRANO, 1942 a), suggesting that manufacture of true *Kamaboko*-forming fish-powder may not necessarily be impossible.[1)]

Anyway, Dr. TAMARU's procedure abovementioned had to be reformed. Spray dryer, which may be similarly expensive as freezer, would not be able

1) EDSALL (1930) has noted that attempts to remove the water contained in precipitated myosin and dry the protein have invariably failed, unless the protein was denatured in the process.

to be kept in operation without help of the latter in the case of dealing with catch of fish. It would result eventually in a matter of expense to treat cheap manufactures by a spray dryer. Moreover, it might be contradiction to add water to fish intentionally before desiccation of it. After all, the author was driven to stand face to face with such problems as follows:

(1) Under what condition is it possible to prepare true *Kamaboko*-forming fish-powder without fail?

(2) Is it possible to prepare *Kamaboko*-forming fish powder in an industrial scale without any special equipment such as that requires large investment for its construction?

However, in order to solve these problems not exclusively through "trial and fail," the author had to first of all enter into the study of *Kamaboko*, since the question "of what substance and by what mechanism is *Kamaboko* formed" had never been answered in the light of modern science until he published a preliminary report "Über die Gelatinierung des Fisches" in 1939 (HIRANO, 1939 a). As it has been proved first by the present author (HIRANO, 1942 b, c, d,), myosin is the unique protein fraction in fish that can form *Kamaboko*.

Although the present work, which will be entered into details below, is not different so far from a crude descriptions of trials performed in order to disclose the behaviors of myosin exclusively in respect of its *Kamaboko*-forming ability, the conclusion arrived at through these trials has been confirmed by repeated experiments. The conclusion has been put into application with several sorts of fish to prepare *Kamaboko*-forming dry fish as registered and published in Japanese Patent No. 158070 (HIRANO, 1943) in the war years.

I wish to express my heartfelt gratitude to the late Dr. S. TAMURA, then the Professor of Tokyo Institute of Technology, for his kind encouragements given me to push this work. I am also indepted to Dr. J. SAMESHIMA, then Professor of Imperial University of Tokyo, Dr. M. MIGITA, Technical Officer of *Tokai* Regional Fisheries Research Laboratory, Dr. H. AIKAWA, Professor of *Kyushu* University, and Dr. Y. SUEHIRO, Professor of *Tokyo* University, who were interested in this work and often encouraged me.

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II. ISOLATION OF MYOSIN

II,1. Introduction

In order to interpret *Kamaboko*-forming ability of myosin, it is necessary first of all to take up isolated myosin in our hand.

Before we were driven to stand alone by the world war II, several works had been published wherein some advanced methods to isolate myosin from muscle were described (EDSALL, 1930; SMITH, 1935b; SMITH, 1937). These procedures are of course not strictly identical between the different authors but essentially similar in the point that not a large amount of myosin is to be isolated at a great expense of neutral salt.

Moreover they have been found by the present author to be associated with a few defects such as follows:

First, the isolated myosin is often contaminated with denatured globulin-*X*, especially when precipitated from the concentrated solution. EDSALL (1930) has obtained water-clear solutions of myosin ("muscle globulin") by repetition of a combined process of precipitation by dilution, redissolution, and salting out. SMITH (1935) has prepared a myosin solution through treating crushed muscle first with 0.01 *N* hydrochloric acid, then neutralizing the acid extract, and finally extracting myosin with a buffered salt solution out from the neutral precipitate. It must be admired that these authors have worked out without sparing any pains in their troublesome procedures.

The second fault of these methods was more serious one for the special situation wherein the present author had been placed, who was then urgent to manufacture *Kamaboko*-forming fish powder and was need of numerous experiences on the colloidchemical behaviors of myosin in a limited period. To secure a plenty amount of myosin preparation for the urgent study, the published procedures were not useful on account that they demanded exclusively the material such as at the highest degree of freshness, a great expense of neutral salt, and a large space of processing room accompanied by an enormous labor. Some more convenient and economical methods of isolation of myosin had to be discovered.

The present author, herefore, has proposed a new procedure which will be entered into details below, wherein not only myosin can be handled in a most concentrated condition throughout the process but also the expense of the agents needed may be extreamly reduced in comparison with the earlier works.

II,2. Details of Experiment

II,2,1. Enrichment of Myosin in the Material by Washing.

In order to perform the operations easily in a narrow space with a least labor, the raw material must be of high grade purity. It was attended, therefore, first of all to remove protein fractions other than myosin off from muscle as completely as possible. But, is it possible to realize an attempt like this without giving rise to denaturation of myosin at all?[2] The question can be answered only through experiments. In addition, the agents used in the experiments must be of low cost as well as of high efficacy, considering the case of large scale application of the results.

According to earlier works, myosin may be kept perfectly insoluble in such an aqueous solution as to have pH corresponding to the isoelectric point of myosin. Thereupon, it was examined, first of all, whether all the soluble proteins except myosin can be removed off or not, by repeated washing of muscle with an aqueous solution of a trial composition. Of course, previously chopped fish was used in this experiment.

Any trial solution may be decided first as an useful wash-solution only when the four articles following are alltogether found to be fulfiled with it:

a. All the known sorts of soluble protein except myosin must be found to come into the washing.

b. No flocculation of protein must occur in the first washing before the test of the article *d* below is finished.

c. The quantities of protein in successive washings must approach to zero with progressing order of the washing.

d. Even if the wash residue which has become so as to give no more protein into washing is crushed again and additionally extracted, any more protein must not come anew into the washing. In other words, destruction of the enclosing structure of muscle must be approved to have been completed.

Each one of these articles will be examined below,

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2) Precipitated myosin forms more or less readily a mass of jelly which becomes often such a hardly dispersible substance that peptizing power of the solvent, in which myosin was soluble prevlously, declares itself to be no more effective to cause dispersion of the protein again into solution out from the mass, unless it is assisted by some proper mechanical operation.

It may be dangerous, therefore, to conclude whether myosin has been denatured or not from an apparent result observed when myosin precipitate is treated with a solvent only for a short while without special caution.

Although the term "denaturation" seems somewhat convenient to deal with unknown changes in physical properties of protein preparations, the definition of the term still continues to be a matter of confusion (*cf.* ANSON (1945)).

The present auther is in a conception, at least in the case with myosin, that the term "denaturation" may be formulated more neatly and decidedly on the basis of its activity as adenosinetriphosphatase which is a distinct characteristic of native myosin as first discovered by ENGELHARDT and LJUBIMOVA (1939), whilst in the present work, he is remained standing persistently upon *Kamaboko*-forming ability of myosin.

II, 2, 1, 1. Preliminary trials.

At the start, a few preliminary trials referable to select any applicable extractant were carried out with very fresh muscle of a turbot, which was crushed with a meat-chopper before use.

A portion of the crushed fish was treated first with a dilute solution of hydrochloric acid, ten times the weight of the fish, adjusting the pH to the isoelectric point of myosin, pH 5.5, so as to suppress the dissolution of myosin. The extract was found to contain turbidity from beginning, which somewhat rapidly turned into fine flocculums and precipitated in a meantime. The HCl-solution, therefore, does not satisfy the article *b* above.

Next, another portion of the crushed fish was extracted with cold tap water ten times the weight of the fish, without pH control.[3] The filtrate of the extract was found to be of pH 6.5 and almost water-clear with faint fluorescence immediately after the filtration. However, there appeared turbidity in a while, coagulating gradually into precipitate which was apparently opaquely white, powder-like substance with little tendency to jelly formation. It may be presumedly denatured globulin-*X* reported by WEBER and MEYHR (1933) and by SMITH (1937). Thus, tap water should be also of no use referring to the article *b*.

Finally, 0.02 *M* acetate buffer of pH 7.0[4] was examined with another portion of the crushed fish. This time, a faintly fluorescent, practically clear extract was obtained. It did not give any trace of precipitate for a long time, but produced a fairly large amount of precipitate when it was acidified to about pH 5.0 with HCl or acetic acid, which appeared very close to that of the described globulin-*X*. From the result of the final trial, it has been suggested that 0.02 *M* acetate buffer at pH 7.0 may possibly be a fitting sort of wash-solution to the author's aim.

Whether all the four articles above can be satisfied or not with this buffer solution, will be investigated in the following.

II, 2, 1, 2. Kinds of protein fractions coming into washing.

Whether the article *a* is satisfied or not, that is, whether all kinds of

3) HATAKOSHI (1935) has extracted muscle of a fresh porgy first with water, then with NaOH solution, while she assumed that the fraction extracted with the latter might be myosin. She has little examined, however, whether her assumption was correct or not. Considering from a logical standpoint, at least a portion of her myosin has been treated as a different fraction from myosin, namely as myotelin, later on by KONDŌ *et. al.* (1937).

4) The author has adopted this buffer solution referring to the work of WEBER and MEYER (1933) wherein they have pointed out that in 0.03–0.005 *M* KCl at pH 7 is dissolved globulin-*X* but not myosin.

the soluble protein other than myosin come extracted or not when a crushed fish is washed with the abovementioned buffer, was examined by an experiment such as follows:

Raw material: A turbot (*Paralichthys olivaceus*) captured on the coast of *Chōshi*, *Chiba* Prefecture. Body-weight, 4.47 *kg*; Body-length, 64 *cm*; Sex, ♀.

Preparation of extract (washing): Body-side muscle was separated up from the fish, the raw material, immediately after death. It was first cut into small pieces and then treated twice by an ordinary type meat-chopper, cooled down previously in a refrigerator. 1000 *g* of the chopped fish, corresponding to 210.5 *g* dry matter or 29.42 *g* nitrogen, were extracted with a cold aqueous solution prepared by dissolving 2 gramm molecules of sodium-acetate in 8 *l* of ice-cold water, which had been kept in a wide-necked glass bottle of 10 *l* capacity. The extraction was carried out adjusting pH to 7.0 and total volume to 10.0 *l* by addition of proper amounts of *N*-NaOH (*ca.* 50 *cc*) and of ice-cold water. After occasional stirring and subsequent still-standing, the mother liquid was strained by suction first through a piece of cotten tissue, next through sheets of filter-paper, yielding 8.96 *l* filtrate. The filtrate, namely, the extract or the washing, of the first order thus obtained was faintly fluorescent and tinged with golden yellow. It contained 99.16 *mg* N per 100 *cc*. Hence, total N of the filtrate, *i. e.*, N extracted by the single operation amounted to 8.885 *g*, corresponding to 30.20 per cent of total N of the chopped muscle. Although the extract remained virtually transparent on that day of preparation, Feb. 16, 1943, it appeared faintly turbid even if without any flocculum on Feb. 22 when the coagulation analysis below was carried out. It was used as the test-solution.

Heat-coagulation analysis I: A portion (200 *cc*) of the test-solution was put into a large glass tube dipped in water which had been held in a beaker of 1 *l* capacity. The beaker was heated gradually until the bath-water began finally to boil. Increase in turbidity as well as occurence of flocculation in the test-solution was observed under continual stirring during the rise of temperature, the rate of which was controled nearly to 1.5°C, per minute. In this trial was detected no distinct abruption in flocculation of protein through the temperature range from 38°C. to 80°C., in which all the protein fractions were coagulated exhaustively, as if there were present in the extract only a single fraction as coagulable protein. The final mother liquid became slightly hazy when heated to boil but turned reversibly clear when cooled. Inquiry into the nature of the haze was reserved, however, since it was so minute in quantity (*cf.* the foot note 10).

After all, the result of the present analysis has been negative.

Heat-coagulation analysis 2: Another portion, about 200 *cc* of the test-solution, was first acidified with 20 *cc* of 0.1 *N* acetic acid to the reported isoelectric point of globulin-*X*, pH 5.1. Flour-like flocculums which appeared in a few hours were separated aside by centrifugation, and the supernatant liquid containing as yet not a small amount of protein was analysed by a classical method of fractional heat-coagulation in a similar manner as described above.

The protein flocculums mentioned above were found almost, if not perfectly, insoluble in a dilute acid-, alkali-, or salt-solution, remaining as a dense but fragile mass of blownish white color.[5] It may presumably consist of denatured globulin-*X* of which native form is to be a member of the soluble proteins in fish.

The results of the analysis showed that the supernatant liquid contained at least four protein fractions in it, which behaved as follows:

1. The first coagulated completely before the temperature rose to 36°. This fraction existed only in a small quantity.

2. The second coagulated between 55–60°C.; below this temperature range remained this fraction dispersed in a state of strong, but stable turbidity such as to penetrate a sheet of filter-paper.

3. The third fraction coagulated between 67–75°C. It was found to exist only in a minute quantity.

4. The fourth was not flocculated by heat but precipitated with trichloracetic acid or sulfosaricylic acid.

(The final filtrate at pH 5.1, which was completely freed from all fractions of heat-coagulable protein, was water-clear and did not become hazy on heating to boil, in contrast with the corresponding filtrate at pH 7.0 described above.)

Now, these data, upon themselves alone, fall of course short for fulfilment of the article *a*. It is necessary for it that they are supported by earlier findings of several authors. Fortunately, SMITH (1937), reviewing and discussing the earlier works, several of which being his own, has numerated

5) It was found that the precipitate, form which the mother liquid had previously been strained off, was pertially soluble in a dilute solution of acid, alkali, or neutral salt. The salt solution was slightly turbid, whilst the acidic or alkaline solution were clear.

WEBER and MEYER (1933) have found that dialysis-precipitate of globulin-*X* is partially soluble in a neutral salt solution. But SMITH (1937) describes that acid-denatured globulin-*X* is perfectly insoluble in such a solution.

Since the myosin content in the routine buffer extract is so minute as it will be confirmec in II, 2, 1, 9, later on, the partial dissolution of the precipitate in salt solution may well be considered as an intimation that the precipitate, at least soon after precipitation, is a mixture of all-denatured and non-denatured globulin-*X*.

four fractions as intracellular proteins in rabbit muscle: *myosin, myogen, globulin-X* and *albumin* (*myoalbumin*). By the virtue of his paper, the present author has reached a conclusion that the fraction 1 above may be considered as *solub'e myogenfibrin,* 2 as *myogen,* and 3 as *myoa'bumin.* The fraction 4 does not fully coincide with *myoproteid* reported first by v. FÜRTH (1895) and recently also by MORI and ASAKAWA (1943). It behaves again different from *metamyosin* trivially reported by the present author in his previous paper (HIRANO, 1942 *c*). Its properties resemble rather those of *albumose* (*myoalbumose* of HALLIBURTON (1887)).

In the light of referrence to these earlier works, it has been concluded that the experimental results obtained above point out the ful lment of the article *a* in the case of 0.02 *M* sodiumacetate buffer at pH 7.0 as the wash-solution. A buffer solution like this would be called conventionally "the routine buffer."

II, 2, 1, 3. Spontaneous flocculation of protein in the first washing.

Since it had already been confirmed that no flocculation of protein took place in the routine buffer extract prepared in II, 2, 1, 2 at least for several days, which was proved to be a sufficiently long period to accomplish the whole course of successive washing of crushed fish, there may be no more need for any intentional experiment to examine whether the article *b* is fulfilled or not with any sort of wash-solution.

In other words, the article *b* is also to be satisfied with the routine buffer.

II, 2, 1, 4. Quantities of protein extracted in successive washings.

It was examined with the routine buffer whether the article *c* can be satisfied or not, namely, whether quantities of protein extracted in successive washings do or do not approach to zero with progressed order of washing. The experiment was carried out as follows:

Table 1.

Orde of extraction	Volume of sample analysed	N-amount measured	N-amount per 100*cc* of extract	Total volume of extract	Total amount of N extracted
	cc	*mg*	*mg*	*l*	*g*
No.1	50	49.582	99.16	8.96	8.885
No.2	200	19.078	9.54	8.90	0.849
No.3	200	7.123	3.56	9.00	0.320
No.4	200	2.143	1.07	9.00	0.096
No.5	200	3.833	1.92	7.93	0.152
Total					10.302

The strained residue from which the routine buffer extract in II, 2, 1, 2 had been separated was subsequently treated with the buffer successively four times, namely, five times altogether from beginning. The amounts of nitrogen found in these successive washings are illustrated in Table 1.

It is expected from the table that the amount of N extractable would perhaps never approache to zero even if the extraction was further repeated. However, already in the third washing was not found such a protein fraction as not to be flocculated by 1/3 saturation with ammoniumsulfate[6]. The slightly solube protein left in the residue after the third extraction, consequently, may be presumably not other than myosin itself, because a minute portion of myosin may possibly go into solution under such a condition as realized by the routine buffer, *i. e.*, ionic strength 0.02 at pH 7.[7]

II, 2, 1, 5. Supplimentary experiment to II, 2, 1, 4.

The abovementioned presumption that the soluble protein contained in the residue after the third extraction may exclusively be myosin, has been supported by results of an experiment wherein another portion of the same chopped fish was repeatedly washed with a highly diluted solution of acetic acid under control of the reaction to the isoelectric point of myosin, pH 5.5. The nitrogen contents, both proteinous and nonproteinous, estimated for each successive washing are shown in Table 2.

It is seen from the table that the amount of protein extracted with such a solution that does not dissolve myosin at all decreases step by step approaching to zero with advance in the order of extraction.

After the eighth extraction where no more nitrogen-extraction was detected, the residue was freed rudely from the mother liquid and crushed by pounding and stamping in a mortar under addition of quartz sand amounting to double the weight of it. Only a trace of protein was extracted from the stamped residue into the nineth extract. In the tenth was found no protein. By the eleventh extraction performed for the residue which had been once more strained and elaborately stamped, non the more protein came extracted anew. These results of observation show that the enclosing structure of muscle has been destroyed completely through the first stamping with quartz

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6) Myosin may be salted out exhaustively form its neutral solution by 1/3 satulation with ammoniumsulfate (SMITH (1935 a)).

7) According to EDSALL (1930), solubility of a certain sample of cow muscle globulin in a potassium phosphate buffer at pH 7.4, O°C., and ionic strength of 0.01 was found to be 0.8 mg N per liter, which is a remarkably smaller value than the lowest value in Table 1. What does account for the probable high solubility of fish myosin remains yet uncertain.

Table 2.

(Total volume of the extraction-system and volume of the washing to be excluded were kept respectively 10 *l* and 5 *l* throughout the repeated extractions. The starting extractive had been washed once before the measurements.)

Order of extraction	Every 250cc of washing at each order contained (*mg*)			$\frac{Pi}{NPi}$ ($i=2,3,4,\cdots$)	Ratio between extracted amounts in each couple of successive extracts		
	Total N	Protein N (Pi)	Non-protein N (NPi)		Total N	Protein N	Non-protein N
1	——	——	——	——	——	——	——
2	8.35	3.74	4.61	0.81	0.629	0.676	0.567
3	5.25	2.53	2.72	0.93	0.480	0.480	0.45
4	2.42	1.21	1.21	1.00	0.50	0.52	0.50
5	1.26	0.66	0.60	1.10	0.59	0.53	0.53
6	0.71	0.39	0.32	1.2	0.62	0.64	0.59
7	0.44	0.25	0.19	1.3	——	——	——
8	——	negative	——	——			

sand, as well as that the soluble protein fractions have already been exhausted from the fish before the tenth extraction.

It is concluded, therefore, that the discrepant tendencies found between the series of values for soluble nitrogen illustrated respectively in Table 1 and Table 2 are to be ascribed to the difference in pH between the two extractants, at a definite value of which myosin may be dispersed to a specific degree.

After all, the routine buffer has been approved to satisfy the article *c* too, at least with regard to the soluble proteins except myosin.

II, 2, 1, 6. Discussions upon the results of II, 2, 1, 5.

Dispite that throughout the washing-operations of the various stages was maintained the relation:

$$\frac{\text{Volume of removed supernatant liquid}}{\text{Total volume of extraction system}} = \frac{1}{2},$$

the ratio between the successively extracted N-amounts fluctuated remarkably around 0.5 as shown in columns 6, 7, and 8 in Table 2, and more pronouncedly for protein-N than for non-proteinous N. It may well be expected that, in such a system as treated above, the extraction-equilibrium is established rather hesitatingly. However, it shouldn ot be always postulated that such hesitating is due to apprehensible short ability of the chopping-machine to affect complete exposure of intracellular protein. On the other hand, the ratio Pi/NPi, *i. e.*, amount of protein-N extracted/amount of non-protein-N extracted,

increases slowly but steadily with increasing order of extraction, *i. e.*, the earlier the stage of extraction, the larger is the proportion of non-protein-N in the extract, suggesting that non-proteinous N comes out into solution more easily than proteinous.

II, 2, 1, 7. Destruction of enclosing structure of muscle.

It might be apprehended that the soluble proteins contained in chopped fish could not be removed exhaustively from the latter owing to possible incomplete destruction of the fine structure of muscle fiber, in which they lie enclosed.

However, it was almost impossible to examine for final residue in II, 2, 1, 4 whether the article *d* could be fulfilled or not, *i. e.*, whether muscular structure of the chopped fish had been completely destructed or not, since the residue contained so much water that it was too slippery to be stamped without addition of quartz sand, while the addition of sand was expected to effect such a condition under which the uniformity of the residue would not be to be assured.

In the trial in this time, therefore, a small portion of the residue left behind the fifth extraction in II, 2, 1, 4 (see Table 1) was put into a 50 *cc*-glass tube for centrifugation. It was crushed by careful stamping under addition of quartz sand, three times by weight of the residue, and exhaustively extracted in a similar manner as reported in the previous paper (HIRANO, 1942c) with the routine buffer instead of the potassium phosphates-potassium chloride solution.

The results of the trial were as follows: while a slightly more nitrogen was found in the first washing from the stamped residue compared with that from the not-stamped residue as a control, the amounts of nitrogen in the subsequent successve washings from the former were found to remain invariable as well as those from the latter. Furthermore, it was verified that an additional stamping of the residue had no effect on the amount of nitrogen, presumably myosin-N, sudsequently extracted.

These results show that the muscular structure can be destructed almost completely by treating fish twice through an ordinary meat-chopper, practically fulfilling the article *d*.

After all, it has been concluded that *all the soluble protein fractions except myosin can be washed off from fish with 0.02 M sodiumacetate buffer at pH 7, if the fish has been treated twice through an ordinary meat-chopper beforehand.*

The final extraction-or wash-residue of chopped fish thus freed from the soluble proteins except myosin would be termed in the present paper "*Bleached*

Fish" for convenience sake, referring to a process termed "*Niku-sarashi*" in Japanese "*Kamabako*" -manufacture.

II, 2, 1, 8. Proportion of soluble protein fractions in fish.

On the basis of the preceding descripion under II, 2, 1, 2, six nitrogenous fractions soluble in 0.02 *M* acetate buffer at pH 7.0 may be numerated in an ordinary fresh fish. They are presumed to be respectively, *globulin-X, soluble myogenfibrin, myogen, myoalbumin, albumose*, and *nonproteinous substance*. In the light of the recent work on native and denatured muscle proteins by SMITH (1937), native intrcellular nitrogenous compounds of radbit muscle may be estimated roughly as follows: (1) *myosin* [47–48%]; (2) *globulin-X* [14–15%]; (3) *myogen* + (4) *myoalunin* [7.14%]; (5) *nonprotein* [15%], neglecting minute pro bable contamination from blood, where the values in brackets, indicating the relative quantities of each classified compund in per cent of total nitrogen of the muscle, are recalculated referring further to his earlier wirk (SMITH, 1934). Turning to fish muscle, it might be supposed to be of an analogous composition if not of identical to rabbit muscle.* The present auther has reported in his previous paper (HIRANO, 1942c) the composition of turbot muscle as follows: *myosin*, 54%; *myogen+globulin-X*, 20%; *metamyosin*, 2%; *indispersible nitrogenous compound*, 9% and *nonproteinous compound*, 15% (intimated). The higher content of myosin in fish compared with mammalian muscle found by the present author is concordant with the earlier findings by REAY and KUCHEL (1936) and by BAILEY (1939).

Now, another example of investigation of fish composition will be given in the following.

Material examined: A portion of the extract under II, 2, 1, 2, was used for analysis. A number of flaky coagulums wers observed, though in a minute qua tity, floating on the surface of the extract, which readily agglomerated into a gelatinous mass entangling each other when the liquid was stirred.[8] Because the amount of this mass seemed so minuted in comparison with the total amount of the protein extracted, it was ejected from the bulk of the mother liquid, and only the latter was used as the test material.

It contained 49.582 *mg* N per 50 *cc*. This value be represented by *p*.

Fractionation of proteins dessolved in the extract: 200*cc* of the test material were put into an ERLENMEYER flask of 300*cc* capacity and acidified to

* *Cf.* DYER, W. J., FRHNCH, H. V., and SNOW; J. M. *J. Fisheries Research Board Can.* 7 (1950) 585. (*C.A.* **44** (1950) 4528 *g*)

8) It is likely that such flaky coagulumus were derived from the fragments of the surface film of the large bubble which was formed at the tip of the filterfunnel when the routine buffer extract of chopped fish was filtered under suction. They were found to be soluble in acid–or alkali–solution and to be precipitated at pH 5.5 from such solutions, suggesting that they were formed of myosin extracted in a minute quantity.

pH 5.1 by addition of 222 *cc* of 0.1 *N* acetic acid. The liquid thus acidified became instantaneously turbid, more strongly with time elapsed, and eventually led to flocculation of protein. However, the mother liquid remained still turbid even after the precipitaion of the flocculum had apparently come to an end; this suggests that the clotting power of the turbidity is rather weak whilst the protein fraction is readily denatured into a state of turbidity. In other words, it may be presumed that the precipitated flocculums are of denatured protein but not of native one. In 2 hours after the acidification, however, distribution of the turbidity in the system became shaded off upwards.

The filtrate from this system was a water-clear liquid tinged slightly with yellow. It contained 41.63 *mg* N per 50*cc*, corresponding to $41.63 \times 222/200 = 46.22$ *mg* N per 50 *cc* of the test material. The latter value be represented by *q*.

A duplicate of such filtrate prepared by a parallel operation gave neither turbidity nor precipitate when tested by dialysis, indicating the absence of globulinous fraction in the filtrate.

On heating this filtrate, no trace of turbidity took place before the temperature reached to 40°C. Nevertheless, there appeared a slight amount of minute fibrous fragments coming out of solution, which aggregated into a cruster on gentle swirling of the system but scattered again spontaneously on rest, behaving as if they were minute needle crystals of a certain organic substance. Futhermore, such fragments were produced steadily increasing in amount through the temperature-range from 20°C. to 50°C., while they began to turn into a turbid suspension when 45°C. was exceeded. Above 50°C., the turbidity became so intense that the system appeared practically opaque. Nevertheless, the turbidity were found of such a character as to pass freely through a sheet of ordinary filter-paper. When the temperature rose to 58°C. or above, however, the mother liquid turned perfectly clear, separating a remarkable amount of coagulums.[9] The filtrate separated from the coagulums contained 23.352 *mg* N per 50 *cc*, in other words, 25.92 *mg* per 50 *cc* of the original test material. This value be expressed by *r*.

When the clear filtrate was further heated to higher temperature, it began to become slightly turbid at about 63°C. The turbidity was subsequently transformed in the meantime into fine flocculums which, growing steadily

9) The aqueous solution of this protein fraction forms, when heated, usually a very stable suspension. Therefore, the fact that the mother liquid becomes clear at such a temperature so as to effect a complete flocculation of it, may suggest that the fraction has been exhausted out of the suspension.

larger and larger with the rise of temperature, were precipitated completely at last when the temperature reached *ca.* 75°C., leaving clear liquid behind. The filtrate of the liquid was found to contain another proteinous substance lacking heatcoagulability, and also found to behave in such a way that it becomes slightly turbid on heating to boil but returns clear reversibly on cooling.[10] There was present in the filtrate 22.77 *mg* N per 50*cc*, corresponding to 25.29 *mg* per 50 *cc* of the original test material. This figure be represented by *s*.

On the other hand, another portion of the initial filtrate at pH 5.1 was treated with 50 *cc* of 10% trichloracetic acid. By filtering off the precipitate then formed, a perfectly clear liquid was obtained. The nitrogen content of this liquid was estimated to be 9.743 *mg* per 50 *cc*. This figure becomes 16.22 *mg* per 50 *cc* referred to the original test-material. The latter figure be denoted by *t*.

Estimation of relative quantity of each nitrogenous fraction: As stated above, the precipitate formed on the acidification of the routine buffer extract to pH 5.1 is received to be *globulin X* in the various points of view. From the results given above, it is obtained per 50 *cc* of the test-material a relation:

$$globulin\text{-}X\ \mathrm{N} = p - q = 3.36\ mg \qquad (1)$$

Now, because the fraction, which coagulates in the filtrate from globulin-*X* when it is heated to 50–58°C., is of such a property as not to be thrown out by dialysis, and further because it agrees quantitatively with the precipitate which formed in another duplicate of the same acidified filtrate

10) This reversible turbidity observed in the present final filtrate at pH 5.1 is apparently discordant with the behavior of the final filtrate at the same pH previously dercribed under II, 2, 1, 2 (Heat-coagulation analysis 2). The nature of this reversible turbidity is not very certain. Albumose might possibly come out of solution reversibly through the probable descension of its hydration caused by boiling-heat, since HALLIBURTON (1887) has denoted that myoalbumose is precipitable by alcohol without being converted into insoluble coagulum. However considering the abovementioned discrepancy observed between the behaviors of the two filtrates at the same pH, the revesible turbidity might more reasonably be ascribed to a possible minute remains of myogen which has escaped from being coagulated. According to WEBER (1925a), myogen is not completely coagulable by heat at pH 5.1 (and also pH>about 7) even at 100°C. in the practical absence of salt.

The present author has also recognized its poor sensibility to heat at pH 5.1 as stated elsewhere. Moreover, the appearance of the said turbidity resembles very closely that of the crystal-like flocculums observed in the filtrate at pH 5.1 under II, 2, 1, 8, when the filtrate was warmed from 20°C to 50°C., namely, directly before the occurence of coagulation of myogen. It may be very probable that such a protein of poor tendency to agglutinate as myogen is thrown out of solution but is not coagulated into an irreversible agglomerate through depressed hydration by heat in the absence of salt (*cf.* also note 13 below).

through "precipitation by disionization"[11] (see II, 2, 1, 9, that follows), it may be considered as *myogen*. Then,

$$\textit{globulin-X}\ \mathrm{N}+\textit{myogen}\ \mathrm{N}=p-r=23.66\ mg \qquad (2)$$

and from (1) and (2)

$$\textit{myogen}\ \mathrm{N}=20.30\ mg$$

The unique protein fraction remaining in the filtrate which was freed from the myogen coagulums is also of albuminous character. It is flocculated completely when the filtrate is heated to 63–75°C., and differs from myogen in the respect that it sustains only difficultly, if it does, "precipitation by disionization." Furthermore, if once precipitated, it becomes insoluble in dilute HCl solutions at pH 2-3. After all, this fraction may be accepted as *myoalbumin* of SMITH (1937).[12] Accordingly.

$$\textit{globulin-X}\ \mathrm{N}+\textit{myogen}\ \mathrm{N}+\textit{myoalbumin}\ \mathrm{N}=p-s=24.29\ mg \qquad (3)$$

By substracting (2) from (3)

$$\textit{myoalbumin}\ \mathrm{N}=0.63\ mg$$

Next, with respect to the non-heatcoagulable proteinous fraction, *i. e.*, *albumose*-like substance,

$$\begin{aligned}\textit{albumose}\ \mathrm{N} &= (p-t)-(p-s)\\ &= s-t\\ &= 9.07\ mg\end{aligned}$$

Finally,

$$\textit{non-protein}\ \mathrm{N}=t=16.22\ mg$$

In the case of the first washing under II, 2, 1, 4, the quantitative composition has been calculated on the basis of the values relating to the total volume of the washing (8.96 *l*), the total amount of nitrogen in it (8.885 *g*), and the amount of each nitrogenous component in an aliquot portion (50 *cc*) of it given above. It is shown in gramm in the second column of Table 3 as well as in per cent of total N of the extractive fish (29.42 *g*) in the third column of the table. In turn, the proportion of these fractions in fish may be calculated as appears in the fourth column of Table 3, assuming that all these fractions have been extracted exhaustively from the fish by the earlier

11) The term "precipitation by disionization" corresponds to that "Entionizierungsfällung" described early by WEBER (1925a). Referring to recent theroies, it may be brought about by refolding of the molecule of protein, which has been denatured by provious molecular unfolding of globulous protein.

12) Metamyosin which has been reported by the present auther (HIRANO, 1942c) to be present even in fish of high freshness would be perhaps misunderstanding of albumose at least in the case with very fresh fish. The former, a sort of derived protein found usually in degradated fish, is of such a character as to be readily salted out with NaCl in a neutral solution. On the contrary, the latter is not salted out in a similar condition.

triple extractions illustrated in Table 1, amounting to 10.054 *g* (sum of the total N extracted in these three washings under neglection of possible myosin contaminations in minute quantities), and further that these successive washings contain the same species of nitrogenous compound in a definite proportion.

Table 3

Forms of nitrogen	Amounts of nitrogen in various forms found in the first washing	Proportions of nitrogen in various forms given in per cent of total N	
		of the first washing	of bleached fish, being calculated on the assumptions in the text
	g	%	%
Globulin-*X*	0.602	2.04	2.37
Myogen	3.640	12.36	14.34
Myoalbumin	0.113	0.38	0.44
Albumose	1.624	5.51	6.40
Non-protein	2.905	9.88	11.46
Sum	8.884	30.17	35.01

When the values in the fourth column of Table 3 are compared with the results for fresh fish of turbot, which have been published in the previous paper (HIRANO, 1942c), several points below become conspicuous:

Even if metamyosin described in the latter is provisionally dealt with as *albumose*, it is remarkably smaller in amount than albumose shown in Table 3. Furthermore, the sum of globulin-*X* and myogen in the table, being 16.00%, is found pretty smaller. It is suspected that, in the previous work, a large portion of albumose if not the entire has been precipitated together with globulin-*X*, myogen, presumably associated with myoalbumin, and remainder of myosin by heat treatment of the filtrate from myosin precipitate under approximate saturation with KCl, thus being estimated as a part of myogen plus globulin-*X*.

Finally, while the amount of non-proteinous N has been intimated to be 15% in the previous work, the corresponding figure in Table 3 is fairly smaller than that, in apparent contradiction to the discussion made under II, 2, 1, 6. However, non-proteinous N is suspected to have been overestimated in the provious work, because of the fact that a larger or smaller portion of proteinous coagulum is often turned soluble by *heating* it in strong acid, accordingly, even in trichloracetic acid solution such as used in the previous work.

After all, these figures listed in the fourth column of Table 3 might

be noting other than of first approximation, since they have been obtained under several assumptions such as described above. It should also be kept in mind that characteristics of metamyosin and albumose (*myoalbumose*) have not yet been fully understood.

II, 2, 1, 9. Confirmation of practical absence of myosin in the routine buffer extract.

That the myosin content of the test material under II, 2, 1, 8 is quite so minute as to be undetectable has been confirmed directly as follows:

100 *cc* of the test material were first acidified by addition of 10 *cc* of 0.1 *N* HCl in order to denature myogen and globulin-*X*. While the acidified system was retained at room temperature for two days, it became gradually turbid, and the turbidity, subsequently turning into flocculums, sedimentated at a slow pace. When the clear filtrate from the precipitate was brought back to pH 6.5–7.0 by addition of dilute NaOH, there appeared a large amount of precipitate, indicating that myogen and globulin-*X* had been denatured. Myosin as well as myoalbumin should not behave like those (SMITH, 1937). That myogen and globulin-*X* had been exhaustively denatured through the first acidification followed by retention for two days was accertained as follows: the precipitate formed by neutralization was filtered off, and the clear filtrate was first acidified once more and then subsequently neutralized.

The system, however, remained perfectly clear whether it was neutralized after heating for a few minutes in boiling water or after reserving for a full day.

In addition, with a duplicate of the acidified test material prepared by a strictly similar process, it was found that the amount of protein N precipitated on neutralization was evaluated as 47.50 *mg*. This figure enables us to calculate the amount of myogen N, referring to the value of globulin-*X* given in (1) under II, 2, 1, 8 (3.36 *mg*/50 *cc*), so as to be 20.39 *mg* per 50 *cc* of the original test material. This last figure agrees very well with 20.30 *mg* which has been obtained previously by heat-coagulation analysis (see II, 2, 1, 8, foregoing).

Now, the final clear filtrate free from myogen and globulin-*X*[13] was found not to produce any more turbidity than a trace of haze on acidification

13) There occurred no reversible turbidity in this filtrate on heating it to boil, not only when it was heated after the pH adjustment to 7.0 but also when it was heated after the acidification to pH 5.1, suggesting that the reversible turbidity treated under note 10 may be certainly due to the possible remains of myogen.

These results of observation denote that myogen might be difficultly coagulated by heat but is easily flocculated by "ionization–and–disionization."

to pH 5.2, indicating that myosin was practically absent in the original test material.

From this result too, it may be proposed that the routine buffer extracts exclusively other fractions than myosin.

II, 2, 1, 10. Inability of myogenfibrin and denatured globulin-*X* to produce *Kamaboko.*

That myogenfibrin as well as denatured globulin-*X* are not able to produce *Kamaboko* by the routine process has been confirmed directly as follows:

When the routine buffer extract of chopped fish was first acidified to pH 3 by addition of HCl and then brought back to pH 6.5 by neutralization with NaOH, there took place a remarkable amount of protein precipitate which was an opaquely white, tinged with brownish yellow, paste-like substance. It corresponds, as described above, to a mixture of myogenfibrin, *i. e.*, denatured myogen, and denatured globulin-*X* detectable by WEBER's (1925a) *Entionizierungsfällug.*

It forms apparently a mass of jelly by the routine process of *Kamaboko* manufacture if it is well strained beforehand. The jelly thus formed, however, is fragile, deficient in elasticity, and may be by no means recognizable as a sort of normal *Kamaboko.*

II, 2, 2. Extraction of Myosin from "Bleached Fish."

"Bleached fish" prepared from ordinary fish, *i. e.*, that of not-highest freshness, is suspected, even after a repeated washing with the routine buffer, to contain, under certain circumstances, some amount of insoluble residue such as derived from myogen and/or globulin-*X* by denaturation. If so, do these residual protein fractions not become extracted together with myosin at all, when such "bleached fish" is treated with a myosin-extracting medium?

According to SMITH (1935b, 1937), myosin is anticipated to be extracted into a neutral salt solution without contamination by myogen and globulin-*X*, if the latter fractions are precipitated by disionization. In the case of that myosin is extracted directly into a concentrated state, however, it becomes usually contanimated with hardly separable suspensions of these denatured fractions owing to the extraordinarily high viscosity of concentrated myosin. Consequently, it is difficult indeed to obtain myosin in a concentrated but clear solution with neutral salt.

To the contrary, as will be described in detail later, the present author has extracted myosin from "bleached fish", therefore, first with a dilute solution of caustic soda. The alkali extract is then acidified to pH 5.5 whereat

myosin is expected to be completely precipitated out of the solution. Finally, the whole system is brought back to pH 8.5 by addition of a small amount of NaOH solution. Only *eumyosin*, the unique *Kamaboko*-forming constituent of fish found by the present author, may be kept insoluble at this pH. It may be isolated at that pH from *pseudomyosin*, a non-*Kamaboko*-forming constituent of fish-myosin in the classical definition. Thus, in the present procedure, eumyosin would be kept in a most concentrated state throughout the process of isolation, except in the course of NaOH extraction, where only a small amount of the agent is consumed at a small expense.

However, in turn, to what extent can the possible remainder of denatured myogen and/or globulin-*X* in bleached fish of common freshness depress the purity of eumyosin precipitate in the present procedure? The problem will be discussed below, standing on the changes in solubility of myogen as well as globulin-*X* in relation to the pH-change by which extraction and reprecipitation of eumyosin are carried out.

II, 2, 2, 1. Contamination with myogen.

Even if myogen was present remaining in bleached fish, it woult not come extracted together with myosin, if the former was not to be dissolved in a weakly alkaline solution such as used for extraction of myosin. On the other hand, even if myogen did come extracted together with myosin, it would not be precipitated in accompany with myosin, if myogen was to remain dissolved in the pH range where myosin is kept out of solution.

It is studied, therefore, how myogen does behave in respect of its solubility when it is brough to the alkaline reaction under which myosin is extracted from bleached fish, and furthermore when the alkali myogen is brought back to the faintly acidic reaction under which myosin is precipitated from its extract.

Test material. 595 *g* of chopped fish of common freshness, corresponding to 15.66 *g* N, were extracted with tap water adjusting total volume of the system to 5.0 *l*. The filtrate of the extract (total volume, 4.47 *l*; pH 6.5) was acidified to pH 5.1 with 20 *cc* of *N* acetic acid. After filtering off the globulin-*X* precipitate, a clear liquid with faint yellow was obtained. This was used as test material. It contained:

total N ..... 13.51 *mg*/20 *cc* (67.55 *mg*/100 *cc*);
protein N ..... 9 584 *mg*/25 *cc* (38.34 *mg*/100 *cc*);
heatcoagulable N ..... 14.16 *mg*/50 *cc* (28 32 *mg*/100 *cc*);
myogen N ..... 9.90 *mg*/100 *cc*

The test material remained clear even after the storage over a month in a refrigerator under addition of toluene. Therefore, it has been concluded

that the test material does not contain globulin-*X*. On the other hand, when 10 *cc* of the test material were mixed into 100 *cc* of distilled water which showed pH value of 5.5, presumably owing to the absorbed CO_2 from the atmosphere, and subsequently stored for several days, there appeared no flocculation in the mixture, suggesting the absence of myosin in the test material.

In contrast with this, when 10 *cc* of the test material were poured into 100 *cc* of fresh tap water, pH of which was found to be about 7.0, there occurred at once a strong turbidity, which was converted, while kept quietly, into a compact heap of opaquely white, powder-like precipitate in a meanwhile. It was apparently quite different from the precipitate of myosin which is usually voluminous and looks rather translucent. It may be considered as a somewhat other fraction than myosin referring to the results of experiment described elsewhere.

Some additional observations on the behaviors of the proteins contained in the test material.

(a) **Salting-out with $(NH_4)_2SO_4$.** A protein fraction such as precipitated by 6/25 saturation with the salt was found present. By 14/25 saturation, all the protein fractions were exhaustively precipitated.[14]

(b) **Coagulation temperature.** Four distinct fractions were found as follows:

1. Soluble myogen-fibrine, thrown out of solution as turbidity between 38–45°C.
2. Myogen, coagulated between 47–58°C.
3. Myoalbumin, coagulated between 65–75°C.
4. Albumose, not-coagulable even at boiling temperature of water.

(c) **Denaturation by acid.** Just as SMITH (1937) has reported on albumins of rabbit muscle, the present test material gave no precipitate upon increasing addition of HCl, but it became turbid at once on the subsequent neutralization with NaOH. The stronger the extent of the previous acidification, the more strongly turbid became the neutralized system. In the case with an intense turbidity, there took place precipitation of protein in a short time.

(b) **Denaturation by alkali.** With increasing addition of NaOH, the test

14) V. FÜRTH (1895) has proposed that the concentrations of ammonium sulfate necessary for complete flocculation of myogen, soluble myogenfibrin, and myosin are respectively > 40%, 26%, and 26%, *i. e.*, nearly full, 3/5, and 3/5 saturation. According to SMITH (1935a), strengths of $(NH_4)_2SO_4$ required to precipitate myogen and myosin have been described to be respectively 2/3 saturation and 1/3 saturation.

material became first weakly turbid at pH 6, indicating the presence of soluble myogenfibrin, then hazy at pH 8, and finally again clear at pH 10 and above.

On bringing back the final alkaline solution step by step to a lower pH with increasing addition of a dilute HCI solution, the system became again hazy at pH 8, but somewhat strongly turbid at pH 6, and remained so at the initial value of pH. 5.1, in discordance with the original appearanc of the test material. The more strongly was alkalinized the test material, the larger portion of the protein was precipitated on neutrazation. The precipitate is recognized as denatured myogen referring to the discriptions (a), (b), and (c) above.

Solubility of myogen denatured by alkali. From the preliminary observations on denaturation of myogen by alkali described above, it is anticipated that the author's procedure for isolation of myosin, where alkali extraction of bleached fish and subsequent precipitation of myosin by neutralization are recommended, can not be performed theoretically in success unless the bleached fish is completely free at least from myogen.

However, in the case with fish of ordinary freshness, it may be difficult to prepare such ideal bleached fish as completely free from myogenfibrin by washing with the routine buffer, for the latter brings back the pH of fish which would have been shifted to an acidic reaction after death of the animal, causing precipitation by disionization of the protein. However, if the amount of myogenfibrin contamination was sufficiently small so that it could be neglected compared with the total amount of myosin, the procedure may be of sufficient use to obtain such a preparation as to fit for studies of various properties of myosin. Hereupon, the author has examined first how much portion of myogen is precipitated on neutralization of its alkaline solution as described below.

Duplicate series of samples were prepared as follows:

Every 20 *cc* of the test material containing 1.98 *mg* myogen N were made alkaline to pH 9.80, 10.70, 11.16, 11.60, and 12.00 respectively by addition of previously determined amounts of 0.1 *N* NaOH, and acidified next day to pH 5.5 with 0.1 *N* HCl.

For one series of the samples, the amounts of the precipitated nitrogen at pH 5.5 were determined and expressed in the percentage of total myogen N as shown in the third column of Table 4. The other series was left standing over a night at pH 5.5. The next day, the samples were brough back to pH 8.5 and filtered after several hours. The amounts of non-filterable N were determined and evaluated further in the percentage of total myogen N (see figures in the fourth column of Table 4).

Table 4.

Number of sample	pH after addition of NaOH	Flocculated myogenfibrin in per cent of total amount of myogen	
		at pH 5.5	at pH 8.5
1	9.80	0.7	3.1
2	10.70	29.5	5.1
3	11.16	46.3	6.3
4	11.60	67.5	8.8
5	12.00	93.4	5.7

These results show that the portion of myogen precipitated at pH 5.5 is not small at all, but it is remarkably reduced when the pH is brought back to pH 8.5, where at least above 90% of myogen is dissolved again, although the figures in the table may be only of poor accuracy on account of the too low concentration of myogen in the test material. Consequently, if the remains of myogen in bleached fish is not of a large quantity compared with that of myosin as expected to be so, the contamination of myosin with denatured myogen, *i. e.*, myogenfibrin, should by no means attain to a significant degree.

Possible extent to which myosin is contaminated with myogenfibrin. Now, the residual amount of myogen in such an ordinary preparation of bleached fish as obtainable without any special caution has been estimated as follows:

A small portion (10 *g*) of the extraction residue from the test material just mentioned above was put into a centrifugation glass tube of 100 *cc* capacity, and extracted repeatedly with a relatively large amount (90 *cc*) of 0.6 *M* KCl solution beffered with potassium phosphate as described in the previous report (HIRANO, 1942b, c), but without sand, until no more protein, chiefly myosin, came apparently into the extractant. Thus by using a relatively large amount of extractant, it may be expected that the possible loss of myogenfibrin through a state of suspension into the extract is avoided, referring to the proposition of SMITH (1935b, 1937). Finally, the residue was stamped thoroughly with addition of quartz sand, and exhaustively extracted. It was found that about 85% of N was extracted from the bleached fish with the buffered KCl solution, suggesting that about 85% of total N in the bleached fish was myosin N.

After myosin in the bleached fish was exhausted, the final residue was extracted three times with 0.01 *N* NaOH under centrifugation and the total N thus cumulatively extracted was estimated as myogenfibrin N; this estima-

tion should be, in the strict sense, related to the sum of myogenfibrin N, possible denatured globulin-*X* N, and further of denatured myosin or myosinfibrin N if such was present. As the result of such an estimation, the upper limit of myogenfibrin content was found to attain to 0.86% of the total N contained in the original bleached fish. True myogenfibrine content of an ordinary bleached fish, therefore, should not exceed at most 1% of myosin content in it.

On the other hand, only a portion less than 10% of myogen may be precipitated at pH 8.5 as illustrated in Table 4. After all, it is concluded that, at pH 8.5 where final isolation of eumyosin is to be performed according to the author's procedure, the contamination with myogenfibrin should not exceed an order of 0.1–0.01% of myosin, even when the eumyosin precipitate is not washed subsequently.

In other words, it may be practically possible to prepare myosin free from myogenfibrin by the procedure of the present author.

II, 2, 2, 2. Contamination with globulin-*X*.

The author's procedure might result in fail if denatured globlin-*X* behaved in the same manner as myosin. Whether such objection does occur or not has been examined as follows:

Preparation of globulin-*X* suspension. The precipitate formed on the acidification of the routine buffer extract to pH 5.1 to prepare the test material under II, 2, 2, 1 was the starting material in the present experiment. The precipitate was filtered after a night, taken apart from the filter paper with shoot of water from a wash-bottle under simultaneous scratching off with a blush, and collected into a 2 *l*-beaker. The precipitate was readily suspensible in water by mechanical agitation. While it formed stable suspensions outside the pH range 4.5–6.5 in the absence of salt, it sedimentated slowly at pH 5 into a heap of fine flocculums which could be dispersed again on stirring the system into an unstable suspension rather easily. The layer of precipitate looked compact, white, and opaque. Because the filtrate from the precipitate, *i. e.*, the test material under II, 2, 2, 1 remained perfactly clear over a month, the precipitate must be of a different protein fraction from those remaining in the filtrate. It is considered to be a precipitate of denatured globulin-*X*.

After the precipitate was washed thoroughly with dilute acetic acid at pH 5.1, it was brought into water of which reaction was adjusted approximately to pH 10 by addition of NaOH. There resulted a somewhat unstable suspension of globulin-*X*, from which, while kept quietly, a brownish white layer of powder-like precipitate was separated out gradually leaving a clear

supernatant liquid behind. When stirred, the system was readily turned back into an apparently uniform suspension which was used as the starting material in the following experiment.

Solubility of globulin-*X* in NaOH solution. Even if it happened that a portion of denatured globulin-*X* is left in bleached fish, it can not amount to above 1% of the total N of the latter as expected from the measured value for the possible contamination with myogenfibrin (0.86% of the total N) in an ordinary bleached fish. Furthermore, if denatured globulin-*X* remained insoluble in such alkaline solutions as those used for extraction of myosin, it will not interfere the isolation of myosin. In such a point of view, the solubility of denatured globulin-*X* in NaOH solution has been investigated as follows:

Starting with the abovementioned unstable suspension of globulin-*X* at pH 10, several samples with varying pH were prepared. They were adjusted in total volume to a definite value (400 *cc*) and kept at room temperature over a night. After filtration, the proportion of dissolved nitrogen was determined for each filtrate. The results are shown in Table 5.

Table 5

Number of sample	To every 350 *cc* of the starting suspension were added (*cc*)		pH of sample	Total N in *mg* per 50 *cc* of sample	Dissolved globulin-*X* in	
	distilled water	0.1 *N* NaOH			*mg* per 50 *cc* of sample	per cent of total N
1	45.0	5.0	9.76	5.306	1.314	24.8
2	40.0	10.0	10.34	5.311	1.725	32.5
3	30.0	20.0	10.90	5.260	2.875	54.5
4	10.0	40.0	11.24	5.440	3.962	74.4

As expected from the figures given in the table, a significant portion of denatured globulin-*X* will become extracted together with myosin into NaOH solution. The stronger the alkalinity of the extractant, the larger portion of globulin-*X* will come out from the bleached fish.

Behavior of globulin-*X* on neutralization of its alkaline solution. If denatured globulin-*X* extracted with alkaline solution can be kept dissolved when the extract is brought to pH 5.5 or subsequently back to pH 8.5, its seperation from myosin may be easily done. Hereupon, the behavior of denatured globulin-*X* with such pH change was followed on the duplicates of the four samples treated in the previous examination. The results are illustrated in Table 6, indicating that: while a considerable portion of globulin-*X* is precipitated on acidification to pH 5.5, the precipitate there formed dissolves again almost completely on subsequent being brought back to pH 8.5.

Table 6

Number of sample	Total N in *mg* per 50 *cc* of sample	Globulin-*X* N					
		at pH 5.5			at pH 8.5		
		per 50 *cc* of sample		Precipitated amount in per cent of total N	per 50 *cc* of sample		Precipitated amount in per cent of total N
		dissolved (*mg*)	precipitated (*mg*)		dissolved (*mg*)	precipitated (*mg*)	
1 a	1.314	0.958	0.356	27	...	...	...
1 b		...	...	...	1.354	−0.04	0
2 a	1.725	1.156	0.569	33	...	...	...
2 b		...	...	...	1.760	−0.04	0
3 a	2.875	1.588	1.287	45	...	...	...
3 b		...	...	...	2.86	0.02	0
4 a	3.962	1.679	2.283	58	...	...	...
4 b		...	...	...	3.675	0.29	7

Although a few per cent of globulin-*X* may possibly remain insoluble even at pH 8.5 when pH of the original extract is higher than 11, the amount of such insoluble part may be quite negligible compared with the total amount of myosin contained in bleached fish from a similar point of view as discussed previously on myogenfibrin.

After all, it is concluded that globulin-*X* too becomes by no means a serious impurity against isolation of myosin according to the authers procedure.

II, 2, 2, 3. Preliminary examination on myosin-extractant.

In the respect to economy, alkali and acid may be certainly preferable to neutral salt to isolate myosin from fish. But if chopped fish is treated first of all with alkali, the subsequent isolation will not be able to be performed with acid only, since the denaturation products of myogen and globulin-*X* behave likely as myosin in respect to their solubility in the absence of salt as stated above. Therefore, myosin should be extracted with alkali first after myogen and globulin-*X* has been excluded away from fish as completely as possible. Under such an idea, the present author has prepared such bleached fish as almost completely deprived of these protein fractions by repeated wsahing of chopped fish with 0.02 *M* sodium acetate buffer at pH 7.0, *i. e.*, the routine buffer. Such bleached fish would be called "normal bleached fish" from a conventional reason. Normal bleached fish may easily be prepared even from fish of common freshness as it is seen from the preceding result (see II, 2, 2, 1).

Myosin is now to be extracted from normal bleached fish with a dilute solution of NaOH. However, is this all that is necessary to extract myosin in a pure state?

Before the question is answered, the reasons should be elucidated why the author recommends an aqueous solution of NaOH as an extractant for myosin, in preference to that of acid, for instance, HCl. The results of an examination that follows will account for one of the important reasons.

Efficacy of HCl-or NaOH-solution as myosin-extractant. Extraction of myosin at various pH has been comparatively observed, while a definite small quantity of normal bleached fish was put into each one of large glass-stoppered test-tubes, and treated with HCl-or NaOH-solution of varying concentration, controling the total volume to a definite value. The relative amount of myosin precipitated on neutralization of the extract varied in such a manner as follows:

(a) In the case of a single extraction, even from fish of highest freshness, the amount of myosin extracted with NaOH solution is far greater than that with HCl solution. Furthermore, such difference increases with decrease in freshness of the extractive.

Such variation in apparent solubility of myosin effected by NaOH or HCl may be due to the difference in peptizing power against myosin gel between these two agents. If bleached fish of highest freshness was processed by "exhaustive extraction," the same result would be attained with either of them. Nevertheless, NaOH solution seems to be more convenient extractant even from the single reason that the troublesome process of exhaustive extraction may be simplified with it more greatly than with HCl solution.

(b) In the pH range of 2 to 12, with HCl solution as well as NaOH solution, more myosin is extracted with a stronger solution. However, the extreme concentration of either solution or the maximum amount of either agent required for the greatest extraction in this pH range is almost identical.

Based on the findings in these preliminary tests, the author has preferred NaOH to HCl as an extractant-agent for myosin. NaOH is not only a very popular and cheep sort of alkali but also may be used without any trouble in nitrogen estimation by KJELDAHL method.

Inhomogeneity of myosin extractable with NaOH Solution. The described relation, that the more strongly alkaline the extractant, the more myosin is extracted from bleached fish, can not be interpreted of course without considerations on the structure of myosin molecule, for the relation does not neccessarily intimate that: the bleached fish contains a numerous

sorts of myosinous fractions having such solubilities as to vary by small extents between each other of them, while different one of these fractions, in turn, comes into solution depending on a momentary condition of the environment in an "all-or-none" fashion.[15]

On the present knowledges in this field, myosin in an aqueous solution of alkali may be regarded as an alkali salt belonging to a special class of polyvalent carboxylic acid, a salt of *myosinic acid* so to speak, in the molecule of which number of the salt-forming groups, being established through proton release of NH_3^+ groups, varies with pH of the environment, but not as a mixture of numerous members of high-molecular carboxylic acids of common type, alkali salts of which are usually far more soluble than their free acid forms. A numerous kinds of such salt may be formed between myosin and alkali at different degrees of neutralization. These alkali salts of myosinic acid may probably have different solubilities each other, while their solubility may well increase primarily with increasing number of net carboxyl groups. Thus it may be concluded that enough addition of alkali so as to effect a high pH leads to elevation of solubility of myosin, promoting the formation of myosinate of high valency.

In turn, with regard to elemental molecules (composing the micell) of myosin, ammonium groups such as release proton more readily may lead to a stronger hydrophilic tendency, fixing the added alkali more intimately and thus causing a more pronounced depression of pH in the environment, compared with those in the same molecule such as only hardly proton-separating and accordingly liable to be subjected to hydrolysis. In other words, at a definite pH of the environment, more easily proton-releasing NH_3^+ groups may tend to effect higher affinity to water, promoting dispersion of the molecule or the micell into solution.

Because there are no reason, however, that ammonium groups with poor proton-releasing tendency and those with strong one should have to belong respectively to different fractions of myosin, it is quite impossible to predict whether myosin might be a mixture of protein fractions or not, even in such a simple case as that it is really composed of only a few fractions such as easily separable from each other, only from the results of observations wherein the change in solubility of myosin is followed as pH of the environment is altered. However, if distinct differences are found between the in-

15) Referring to the modern conceptions in this field, it may be considered that myosin lies in bleached fish forming micelles through association among its molecules. Accordingly, dispersion or dissolution of myosin from bleached fish into an aqueous solution may depend upon the peptization equilibrium between such micelles and the solution. If the equilibrium is shifted so as to cause an increased peptization with increased alkalinity, the more myosin would come into solution, the more strongly alkaline solution is used as the extractant.

trinsic properties of myosin preparations, which have been at first extracted from the same bleached fish at different pH and then brought into a definite environment wherein they are compared with each other, it might be decided that myosin in the bleached fish is not homogeneous;[16] of course great caution should be needed to attain an objection-free conclusion, since a strongly alkaline solution of myosin is somewhat unstable so that its physical properties often change with time elapsed even kept isolated from the outside and in cold.

As one of the intrinsic properties to be compared between myosin preparations provided at different alkalinities, the author has chosen "*Kamaboko*-forming ability" at such a pH as to agree with the isoelectric point of myosin (pH 5.5), on account that the final aim of the present research is to acquire a fundamental upon which the manufacturing technique of "powder of raw fish" might be established. Although, as reported previously (HIRANO, 1942b), this ability can be measured quantitatively only with some difficulties, it may be estimated qualitatively by a sensory test with passable exactness.

It has been experienced by the author invariably that, whilst a fraction of myosin such as extracted from normal bleached fish with NaOH solution at pH *lower* than about 9 is quite deficient in *Kamaboko*-forming ability, another fraction extracted at pH *higher* than about 10, despite of the suspectable instability in such a high pH region, has a prominent ability producing *Kamaboko*. It has been also confirmed that the former fraction never acquires newly *Kamaboko*-forming ability by bringing it once to pH above 10, but that the latter becomes never deprived of the ability by bringing it to pH below 9.

From these observations, it has been concluded that there are present in bleached fish two kinds of myosin which differ from each other at least in respect of *Kamaboko*-forming ability.[16] They can be separated from each other on difference in solubility in the pH range of 9 to 10. From conventional reason, the author has named the fraction without *Kamaboko*-forming ability "pseudomyosin," while the fraction with the ability "eumyosin."

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16) KONDO, *et. al.* (1937) have prepared a protein fraction from a lobster (*Penaeus canaliculas Oliv.*) by extracting the muscle with an aqueous solution of NaOH and named it "myotelin" which they consider as a different fraition form myosin. The present author, however, has found that myotelin fraction obtained by the published procedure from a turbot behaves just alike classical myosin at least in respect of Kamaboko-forming ability.

It should be also noted that only a minute portion of alkali-soluble protein would remain if fish is extracted beforehand *exhaustively* with salt solution (*cf.* II, 2 2, 1).

II, 3. Procedure for Isolation of Eumyosin.

II, 3, 1. Preliminary Observations.

When the normal bleached fish prepared from a highly fresh raw material is extracted with an aqueous solution of NaOH at pH of about 11, a clear solution tinged with yellow is obtained after filtration of the mother liquid. The said solution is of a high viscosity and strongly light-scattering. During the filtration, there was often formed a large bubble hanging at the tip of the funnel, which, after ruptured, was transformed into an icicle-like, small column of transparent jelly.

On addition of HCl to the filtrate by small amounts, it behaves as follows:

The clear solution becomes fluorescent in the neighborhood of pH 9. When the reaction approaches pH 8.5, there appears first a slight turbidity throughout the bulk of the solution, which, after a moment, turns almost suddenly into clouds of coarse flocks which sedimentate rapidly to the bottom of the vessel. The layer of the precipitate there formed looks semitransparent, bluish green with reflected light, and reddish brown with transit light, while the supernatant liquid left behind appears transparent but exhibits a faint TYNDALL-cone. At pH of about 7, gentle agitation of the system enables the sediment layer to disperse again uniformly throughout the system, yielding a thick suspension with a semitransparent, bluish white appearance. On acidification to pH of about 5.5, intense flocculation of protein occurs again forming this time a layer of settlings of fine flocculum, which appears semitransparent, violet-tinged white. The supernatant liquid becomes perfectly water-clear at this pH. The sedimentation of flocculums, however, proceeds *less* speedily at pH 5.5 than at pH 8.5. In the acidic range beyond pH 5.0, the system becomes a homogeneous, fluorescent solution.

If the reaction is brought back to pH 7 by addition of a small amount of NaOH after the precipitate is kept at pH 5.5 for a while, the whole system turns once more into a fairly stable, weakly turbid, uniform suspension; in turn, the turbidity looks somewhat stronger than before. On further bringing back the pH up to 8.5, there again bursts out a dense cloud of flocks apparently in larger quantity this time than at the first time, separating rapidly a clear supernatant liquid wherein only a small quantity of protein remains dissolved.

Protein in the supernatant at pH 8.5. If a portion of the supernatant liquid at pH 8.5 is taken up and brought once more to pH of about 5.5, not a small amount of protein comes slowly out of solution forming a deposit-heap of fine flocculums, which appears with reflected light translusent, tinged

with blue. The precipitation seemed to be performed most precisely at pH 5.2 leaving supernatant liquid perfectly deprived of protein. On being concentrated by centrifugation, the deposit set into a mass of jelly which looked quite different from the mass of denatured globulin-*X*. Addition of common salt to the jelly led to formation of a sticky paste which set anew into a mass of jelly before long. A number of trials to trun the pastes into *Kamaboko* have invariably failed, notwithstanding that the proportions of the salt-addition are varied over a wide range of composition, as well as that the protein contents are elevated to varying extents by straining the pastes with dry filter-paper, while the seeming masses of jelly have got constantly out of shape on steaming.

On the other hand, a concentrated solution of this protein fraction at pH 7–8 showed a distinct flowbirefringence when tested by ZOCHER's (1921) method. Furthermore, stable filaments (tensile strength in dry $\fallingdotseq 10\ kg/mm^2$) have been obtained with it, by extruding it through an injection-needle into a coagulation bath containing an aqueous solution of sulfosaricylic acid, and subsequently, stretching, washing, and drying the resultant filaments.

Notwithstanding that this protein fraction resembles closely myosin described in earlier publications, it differs entirely from myosin as far as it concerns *Kamaboko*-forming ability.

Protein in the precipitate at pH 8.5. Now, if the precipitate at pH 8.5 is brought again to pH 5.5, concentrated by centrifugation, and mixed with a small quantity of common salt, it acquires an enormous viscosity such as that of rice cake just made by pounding. The sticky paste thus formed sets into a mass of elastic gel in time, which can be turned by steaming further into a mass of typical *Kamaboko* that behaves conspicuously elastic. Thus, typical preparations of *Kamaboko* have been obtained invariably without fail from the precipitate at pH 8.5.

When treated with increasing amounts of an aqueous solution of NaOH, the entire precipitate was liquified at last at pH higher than 11 into a viscous solution associated with a liquid elasticity, which exhibited a marked flowbirefringence and was easily spun into a stable filament. At neutral pH, the precipitate was dispersed uniformly into 1 *M* NaCl solution yielding a fluorescent, semitransparent sol.

Thus, the protein fraction agglutinating at pH 8.5 not only coincides with reported myosin perfectly as far as examined (*cf.* Part II) but also possesses a marked *Kamaboka*-forming ability. It would be named "eumyosin" hereafter in the present paper.

II, 3, 2. Isolation of Eumyosin.

Standing on the results of the preliminary observations just stated above, the author has adopted a procedure to isolate eumyosin, *i. e.*, the unique *Kamaboko*-forming component of fish, such as follows:

Normal bleached fish is put into cold water of 20 times by weight of the former and stirred thoroughly from time to time, while the pH of the system is adjusted to about 12 by addition of *N* NaOH solution. After the system is kept in a refrigerator over a night, the supernatant liquid is siphoned up and filtered by suction first through cotten cloth, next through sheets of filter paper. The filtrate is then acidified to pH 5.5 by addition of *N* HCl solution and subsequently kept quietly over a night under addition of a few drops of toluene. The next day, pH of the system is brought back to 8.5 by addition of *N* NaOH solution, whereby eumyosin is agglutinated into coarse, snow-flock-like aggregates. After the aggregates have settled completely, the supernatant liquid is siphoned off. The precipitate is then washed repeatedly with a highly diluted NaOH solution under controling the pH to 8.5, while the total volume of the system is also kept constant. At the same time, a small definite amount of each successive washing is taken aside in order to estimate the protein content of the washing comparatively referring to the intensity of flocculation caused by addition of a definite quantity of 10% sulfosaricylic acid. The precipitate is washed repeatedly until no more decrease in intensity of flocculation is detected for the successive washings so that pseudomyosin may be removed off as completely as possible. The precipitate is further washed with tap water and finally with distilled water until Cl^- ion becomes indetectable in the washing. After all, a small amount of toluene is added to the system if necessary.

The eumyosin precipitate thus purified is a semitransparent, paste-like substance. It gelatinizes spontaneously into a mass of jelly in a long time, which can be dispersed readily throughout the bulk of water by mechanical agitation yielding a quasi-homogeneous suspension somewhat unstable.[17]

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17) The author's interpretation given for gelation of fish in his preliminary report (HIRANO, 1939 b) is by no means satisfactory.

However, the observation stated here that when isoelectric precipitate of myosin is brought to pH 8.5 whereat the protein may certainly become hydrophilic through formation of alkali-myosinate, eumyosin agglutinates readily into such deposit as to set spontaneously into a thixotropic mass of jelly, suggests that: the gelation of fish, *i. e.*, the so-called "*suwari*" phenomenon detectable in *kamaboko*-manufacture may possibly result from development of hydrophilic tendency, *i. e.*, increase in hydration of myosin caused by the addition of common salt which is a wellknown peptizing agent for myosin. Since the water content of fish may be usually far too small to effect appreciable dissolution or dispersion of myosin, referring to EDSALL (1930) who has pointed out the low concentration on protein in a possible preparation of the muscle globulin sol, water may be fixed exhaustively by myosin thus reduced hydrophilic by NaCl, leading to gelation of the system as a whole but not to liquefaction, just like that a small addition of water to powder of anhydrous sodiumsulfate leads often to formation of a block of GLAUBER salt but not to an aqueous solution of the salt.

The effect of the minute addition of NaOH upon the behavior of nearly isoelectric eumyosin, which is observed in the present procedure, is of importance in the respect that it suggests an agreeable explanation for the motive of the so-called "*suwari*" phenomenon. Relating to the whole process of *Kamaboko*-formation, main parts of which might be considered to be composed of (1) peptization stage of rather hydrophobic aggregate of myosin by added NaCl, (2) gelation stage of rather hydrophilic suspension of the peptized myosin through its hydration, and (3) setting stage of the hydrated gel of myosin by heat, colloidscientific considerations will be entered into in the later paper.

II, 3, 3. Some Observations on the Behavior of Pseudomyosin.

Separation and purification of pseudomyosin. As stated previously, the protein in the supernatant liquid separable from the eumyosin deposit at pH 8.5 may be composed mainly of pseudomyosin. If the supernatant is acidified to pH 5.2, all the protein contained will be thrown out of the solution. The precipitate thus formed will not dissolve so entirely as before if it is brought again back to pH 8.5, while a minute portion of it will be left insoluble as gelationous flocks. Since the insoluble flocks were found to behave just like as those of eumyosin in respect of *Kamaboko*-forming ability, it may be considered that they are not other than those of eumyosin-remainder which has escaped the preceding agglutination at pH 8.5.

Such pH fluctuation as 8.5⇄5.5 is repeated until no more protein remains insoluble at pH 8.5, while the precipitate at pH 8.5 is filtered off each time if it is formed. Finally, the clear solution at pH 8.5 is acidified to pH 5.2 and the precipitate of pseudomyosin there formed is purified by washing thoroughly at pH 5.2 with a dilute solution of HCl.

The preparation of pseudomyosin thus purified appeared seemingly as if it were thin rice-gruel.

Solubilities at various pH. Solubilities of pseudomyosin in water were measured at varying pH as follows:

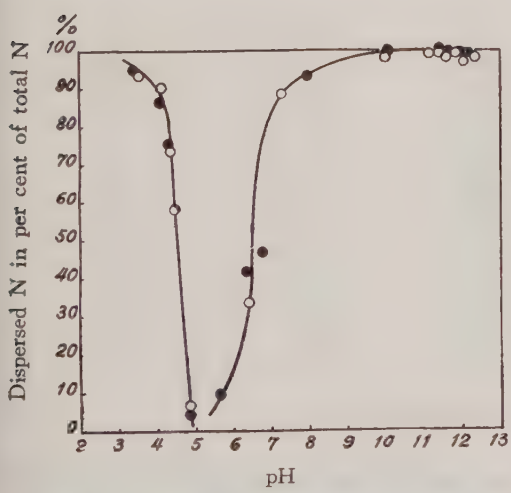

Fig. 1.—Relation between solubility of pseudomyosin and pH. Soft marks, Suspension A; solid marks, Suspension B.

Every 150 *cc* of the suspension of pseudomyosin prepared by such a procedure as described above were added with calculated amounts of distilled water and 0.1 *N* hydrochloric acid or 0.1 *N* sodiumhydroxide so that a series of samples having a constant total volume and varying values of pH may be obtained. For an aliquot portion of each sample was estimated soluble nitrogen, *i. e.*, nitrogen such as that passes through a sheet of common filter paper. The results were recalculated into percentages of total nitrogen. They are illustrated in Table 7 as well as in Figure 1.

It should be noticed that pseudomyosin dissolves above 70% in water at pH 7.

Table 7.

Number of measurement	HCl or NaOH added to 150 cc of the starting auspension (cc)	pH attained	N content per 50 cc of the filtrate (mg)	Dissolved N in per cent of total N
	Suspension A			
1	N/10 HCl, 8.0	3.55	7.83	93.4
2	〃 〃 4.0	4.15	7.57	90.3
3	〃 〃 2.0	4.40	6.16	73.5
4	〃 〃 1.0	4.50	4.87	58.1
5	0	4.85	0.55	6.1
6	N/10 NaOH, 1.0	6.45	2.81	33.5
7	〃 〃 2.0	7.30	7.18	85.6
8	〃 〃 4.0	10.00	8.20	97.8
9	〃 〃 8.0	11.15	8.30	99.0
10	〃 〃 12.0	11.40	8.29	98.9
11	〃 〃 16.0	11.60	8.19	97.7
12	〃 〃 24.0	11.85	8.30	99.0
13	〃 〃 32.0	12.05	8.15	97.2
14	〃 〃 48.0	12.35	8.25	98.4
	Suspension B			
1	N/10 HCl, 8.0	3.42	7.87	95.1
2	〃 〃 4.0	4.10	7.16	86.5
3	〃 〃 2.0	4.32	6.26	75.6
4	〃 〃 1.0	4.83	0.34	4.6
5	〃 〃 0	5.66	0.80	9.7
6	N/10 NaOH, 0.5	6.40	3.45	41.7
7	〃 〃 1.0	6.80	3.85	46.5
8	〃 〃 2.0	8.00	7.72	93.2
9	〃 〃 4.0	10.10	8.28	100.0
10	〃 〃 8.0	11.44	8.30	100.3
11	〃 〃 12.0	11.55	8.29	100.1
12	〃 〃 16.0	11.68	8.25	99.7
13	〃 〃 24.0	*11.968	8.25	99.7
14	〃 〃 32.0	*12.194	8.28	100.0

* These values were determined electrometrically.

Viscosities at various pH. The relative viscosity (η_R) was measured at 25°C. with OSTWALD viscosimeter for the filtrate from each one of the samples prepared just alike those in the preceeding experiment. The results are shown in the fourth column of Table 8. The values of intrinsic viscosity in the usual definition are also given in the fifth column of the table. They are also plotted against pH of the samples in Figure 2.

Table 8.

Number of measurement	pH attained	Protein content in gramms per 100 *cc* of the filtrate (C)	* Relative viscosity (η_R)	Intrinsic viscosity $\left(\frac{log\ \eta_R}{C}\right)$
		Suspension A		
1	3.55	0.1017	1.140	0.560
2	4.15	0.0984	1.133	0.551
3	4.40	0.0801	1.102	0.527
4	4.50	0.0633	1.070	0.464
5	4.85	0.0072	1.003	0.181
6	6.45	0.0365	1.081	0.928
7	7.30	0.0933	1.244	1.016
8	10.00	0.1065	1.109	0.418
9	11.15	0.1078	1.096	0.369
10	11.40	0.1077	1.088	0.340
11	11.60	0.1064	1.085	0.333
12	11.85	0.1078	1.074	0.288
13	12.05	0.1059	1.070	0.278
14	12.35	0.1073	1.061	0.239
		Suspension B		
1	3.42	0.1022	1.155	0.612
2	4.10	0.0931	1.130	0.570
3	4.32	0.0814	1.108	0.548
4	4.83	0.0044	1.003	0.296
5	5.66	0.0104	1.005	0.211
6	6.40	0.0448	1.084	0.782
7	6.80	0.0500	1.102	0.844
8	8.00	0.1003	1.260	0.944
9	10.10	0.1076	1.108	0.415
10	11.44	0.1078	1.090	0.347
11	11.55	0.1077	1.088	0.340
12	11.65	0.1073	1.084	0.336
13	11.968	0.1073	1.076	0.296
14	12.194	0.1076	1.073	0.284

* The measurements of relative viscosity were carried out at 25°C.

As it is seen from these illustrations, intrinsic viscosity of pseudomyosin attains at pH of about 7 such an enormous value, exceeding 1, as that which EDSALL (1930) has found on his muscle globulin of mammals.

Dispersibility in aqueous solution of NaCl. Solubilities, *i. e.*, dispersibilities of pseudomyosin in aqueous solutions containing sodium chloride in varied concentrations were measured as follows:

Every 100 *cc* of the pseudomyosin suspension prepared by the routine procedure described above were added with 5.0 *cc* potassium phosphate buffer

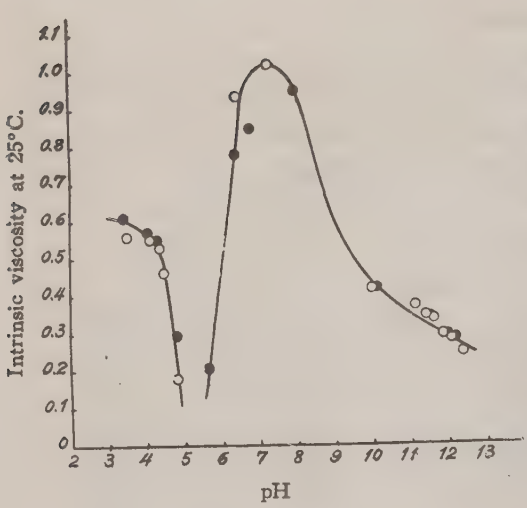

Fig. 2.—Intrinsic viscosity of pseudomyosin at 25°C. in relation to pH. Soft marks, Filtrate from suspension A; Solid marks, Filtrate from suspension B.

at pH 7 (COHN, 1927), calculated amount of distilled water, and 2 *M* or 5 *M* NaCl solution so that a series o samples with a constant total volume (200 *cc*), a constant pH (7.0), and varying concentrations on NaCl may be provided. They were stirred from time to time and filtered after a night when a virtual equilibrium was found to have been established. Dispersed nitrogen was determined for an aliquot portion of the filtrate.

Dispersibility of pseudomyosin may be represented in the ratio of the amount of dispersed nitrogen to that of total nitrogen of the sample. Its values are listed in Table 9 and also shown in Figure 3 correlatively with sodium chloride concentrations in the samples.

Table 9[18]

Number of sample	NaCl concentration (*M*)	Extent of turbidity of the filtrate	pH of the filtrate	N content of the filtrate (*mg*/50 *cc*)	Filterable N in per cent of total N	Amount o protein detected in the filtrate
1	0	−	7.0	0.397	36.6	+
2	0.0625	−	6.9	0.397	36.6	+
3	0.125	−	6.8	0.407	37.5	+
4	0.25	±	6.7	0.459	42.3	+
5	0.5	+++++	6.6	0.777	71.5	+++++
6	1.0	+++++	6.5	0.897	82.6	++++++
7	1.5	+++++	6.3	0.914	84.2	++++++
8	2.0	+++++	6.0	0.835	77.0	?
9	2.38	+	5.8	0.650	59.9	?

18) (1) Total nitrogen content was determined for every 50 *cc* of the suspension which was homogenized by agitation immediately before the sampling. The result obtained (1.085 *mg*/50 *cc*) was evaluated as 100%.

(2) Existence of protein in the filtrate was tested semi-quantitatively by observing the extent of flocculation caused by addition of 5 *cc* of 10% sulfosaricylic acid to every 20 *cc* of the filtrate. For sample No. 8 and No. 9, however, on account that crystals of sulfosaricylic acid together with those of sodium chloride were precipitated leaving a clear liquid behind, the results were obscure. In Table 9, the mark + denotes turbidity of filtrate (in the third column) or flocculation of protein (in the seventh column), number of it indicating their strengths. The sedimentation volumes of the flocculated protein were found to be almost identical throughout all the samples after 2 days. However, the deposit layers in the samples with lower numbers than 4 appeared rather translucent, while those with higher numbers than 5 were of white opacity.

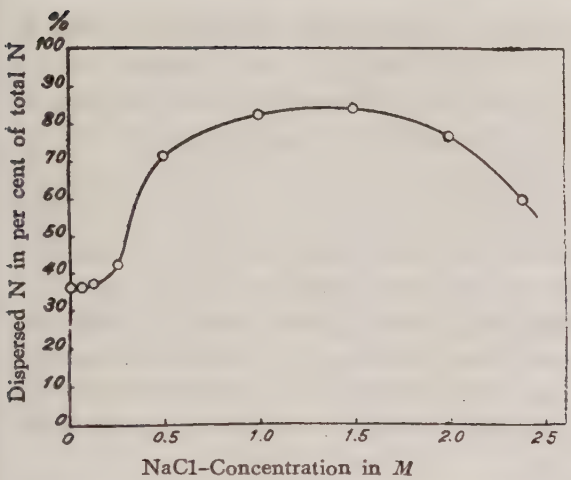

Fig. 3.—Solubility of pseudomyosin in beffered NaCl-solution.

Despite that the portion so dispersed as to pass through filter paper was not measured to reach 100% of the total protein even for sample No. 7 for which the maximum value was found, samples No. 5—No. 8 were remained as stable, uniformly dispersed suspensions against the gravitational force.

It is seen from the results of the present experiment that pseudomyosin is dispersed as far as to nearly 37% in 0.025 *M* potassium phosphate buffer at pH 7 without NaCl (0.15 by ionic strength). In this connection, it should be also remembered that solubility of pseudomyosin in an aqueous solution at pH 7 such as containing a minute quantity of NaOH may attain above 70% as already described (see Table 2 and Figure 1). Dispersibility of pseudomyosin in plain water seems to be depressed by a small contamination of salt.

Salting out with $(NH_4)_2SO_4$. Pseudomyosin dissolved at pH 7.0 is salted out partially by 1/3 saturation with ammonium sulfate and completely by 1/2 saturation with the salt, indicating that the protein may belong to a globulin class. It seems to behave more hydrophilic than the described myosin.

Coagulation by heat. Pseudomyosin in aqueous solution, in the absence of salt, is coagulated by heat in the pH range of 5–6 but not at other pH.[19]

Combination with acid or alkali. Equivalent weight of pseudomyosin as cation or that as anion measured by titration with HCl or HaOH (see Table 7) were found respectively 320 and 250. Standing upon these evaluations and further assuming that the average residual weight of amino acid constitutents of pseudomyosin is 115 (*cf.* Bailey, 1944), it might be inferred that nearly half of the residual groups in the molecule of pseudomyosin are composed of polar groups, the acidic ones being predominant over the basic.

Discussion. Reflecting upon the large affinity of pseudomyosin to water, it may appear somewhat curious that pseudomyosin does not practically come

19) According to WEBER (1925 a), myogen is heatcoagulable in the pH range of 6-7 in a small coexistence of salt but not at other pH (see also II. 2, 1. 2 and II, 2, 2, 1 in the present paper, where it is described that myogen was coagulated by heat at pH 5.1).

into solution, at least in the early stage of extraction, when bleached fish is extracted with the routine buffer, *i. e.*, 0.02 *M* sodium acetate buffer at pH 7.0 (*cf.* II, 2, 1, 9). Presumably, this protein fraction might be either situated in the depth in A band (poriferous sarcous element) of striated muscle or present linked loosely with eumyosin, which is supposed to be a main component of A band, so as to become extracted first when eumyosin is partially dissolved or swollen up through imbibition of NaOH solution into bleached fish.

It might be likely that pseudomyosin agrees very nearly if not perfectly with that curious protein described by EDSALL (1930), which he obtained sometimes when he treated chopped muscle with a relatively small amount of extractant. He states that the said fraction is of lower viscosity, less light-scattering power, and higher solubility compared with the typical muscle globulin.[20]

It might be presumed that, if extraction is carried out in a concentrated condition with a small amount of solvent, only a little portion of eumyosin will be extracted on account of its greater tendency to agglutinate than to disperse, while pseudomyosin with poor agglutinating power will come freely extracted, thus leading to the preponderance of the properties of pseudomyosin in the extract. On the other hand, when a relatively large amount of solvent is used, the charecteristics of pseudomyosin will be overshadowed by those of eumyosin which may be a far more predominant constituent of muscle globulin.

Any detailed relation between pseudomyosin as well as eumyosin and the recently published fractions of muscle protein, *i. e.*, myosin-A as well as actin of SZENT-GYÖRGYI (1947) is not yet certain, whereas pseudomyosin appears to resemble somewhat SZENT-GYÖRGYI's myosin, *i. e.*, myosin-A, in respect of the hydrophily in the absence of salt.

(to be continued in Part II)

REFREENCES

ANSON, M. L. (1945). *Advances in Protein Chemistry* **2**, 361. Academic Press, New York.
BAILEY, K. (1939). *Biochem. J.* **33**, 255.
BAILEY, K. (1944). *Advances in Protein Chemistry* **1**, 302. Academic Press, New York.
COHN, E. J. (1927). *J. Am. Chem. Soc.* **49**, 173.

20) WEBER (1925 b) has also reported, presumably being related closely to the observation of EDSALL, that, when he washed a fresh press-residue of crushed rabbit muscle with *M*/150 acetate buffer or *M*/100 phospate buffer at pH 7–7.5, not a small amount of myosin, the isoelectric point of which was found to be at pH 5.2, came into solution.

EDSALL, J. T. (1930). *J. Biol. Chem.* **83**, 289.
ENGELHARDT, V. A., and LJUBIMOVA, M. N. (1939). *Nature* **144**, 668.
HALLIBURTON, W. D (1887). *J. Physiol.* **8**, 183.
HATAKOSHI, Y. (1935). *J. Chem. Soc. Japan* **56**, 221.
HIRANO, H. (1939 a). *Fishery Investigation* **6**, 63.
HIRANO, H. (1939 b). *Bul. Japan. Soc. Sci. Fish.* **8**, 29.
HIRANO, H. (1942 a). *J. Imp. Fish. Exp. Station* **12**, 56.
HIRANO, H. (1942 b). *J. Chem. Soc. Japan* **63**, 345.
HIRANO, H. (1942 c). *J. Chem. Soc. Japan* **63**, 644.
HIRANO, H. (1942 b). *J. Chem. Soc. Japan* **63**, 1081.
HIRANO, H. (1943). Japanese Pat. 158070
KONDO, K., YAMADA, T., and SHINANO, S. (1937). *J. Agr. Chem. Soc. Japan* **13**, 1129.
MORI, T., and ASAKAWA, S. (1943). *Bul. Japan. Soc. Sci. Fish.* **12**, 67.
REAY, G. A, and KUCHEL, C. C. (1936). *Dep. Sci. Ind. Research. Rept. Food Investigation Board*, p. 93.
SMITH, E. C. B. (1934). *J. Soc. Chem. Ind.* **53 T**, 351.
SMITH, E. C. B. (1934a). *J. Soc. Chem. Ind.* **54 T**, 152.
SMITH, E. C. B. (1935b). *Dep. Sci. Ind. Research, Food Investigation Board*, p. 19.
SMITH, E. C. B. (1937). *Proc. Roy. Soc. London* **B 124**, 136.
SZENT-GYÖRGYI, (1947). *Chemistry of Muscular Contruction*. Academic Press. New York.
TAMARU, S. (1928). Japanese Pat. 76628
V. FÜRTH, O. (1895). *Arch. exp. Path. u. Pharm.* **36**, 231.
WEBER, H. H. (1925a). *Biochem. Z.* **158**, 443.
WEBER, H. H. (1925b). *Biochem. Z.* **158**, 473.
WEBER, H. H., and MEYER, K. (1933). *Biochem. Z.* **266**, 137.

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ON THE NORMAL ANTIBODY IN FISH SERUM

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In the serums of living creatures in general there exists antibody apart from immunity. Though its quantity is little, its protective effect is recognized. It was BUCHNER (1889) that first discovered the bactericidal effect of serum. Since he verified the natural resistibility of a living body to germs and called its cause Alexin, it has come to be regarded as important as white blood corpuscles as a great factor of the protective effect of the host against invading germs. The bactericidal effect of the serum varies according to the kind of germ and animal. F. KAUFFMANN[1] (1926) stated that fresh blood, serum, exudation, etc. have the effect of sterilizing against certain kinds of bacteria or preventing their growth. SCHONBERG (1932) counts the small quantity of natural immune body in fish as one of the causes of its tainting quicker than meat. Kintaro KIMURA[2] (1932) states that it is doubtful whether fish has the same bactericidal substance as meat and that its feebleness is perhaps partly responsible for the quick tainting of fish.

As the result of his studies in the serum of crab by using bac. typhosus, non-liquefacious pseudomonas and Staphyloc. aureus, K. ANGERER[3] (1909) proved that it has a bactericidal effect against each of them, and recognized that it totally extirpated in 21 hours Bac. typhosus, while it showed the most decrease in 7 hours against non-liquefacious pseudomonas and in 3 hours against Stophyloc. aureus, its bactericidal effect abating thereafter with the lapse of time and the bacteria reproducing themselves.

Since warm-blooded animals are thought to have a stronger protective power than cold-blooded ones, I made a comparative test with their serums in order to probe into the fundamental case of the quick tainting of fish.

I. Method of test.

1) Serums of fish used in the test: Carp, Cyprinus carpio Linné was used as representative of fish.
Sea-fishes were also used, but since the test cases are few, they are omitted here. First, the breast is opened and blood is taken from the heart in a bacteria-free manner. The blood is put in a sterilized test tube, which is slanted. After the blood coagulates, it is left

alone in an ice chamber for a night, and serum is collected therefrom.

2) Serums of warm-blooded animals used in the test: Blood is taken from the vein in a bacteria-free manner, and serum is collectedby the same manner as mentioned above.

3) Kinds of bacteria used in the test: Pseudomonas ovalis separated from the flesh of shark. Bacillus Cholerae which was given me by the Government Institute for Infectious Diseases (Inaba strain).

As the quantity of fish serum was small, tests with many strains could not be made, and therefore, 1 strain of non-liquefacious pseudomonas which is related to fish and 2 strains of bacillus cholerae, were used.

II. Agglutination value of each normal serum.

Prior to the test, the agglutination value of each serum was investigated in usual way. Its results are as follows.

Table 1. *Agglutination Value.* Antigen C Cholera; P Pseudomonas.

	Serum	Antigen	Dilutions of Serum					
			12.5	25.0	50.0	100.0	200.0	K
Warm-blooded animals	Human	C	++	+	±	−	−	−
		P	+	+	+	−	−	−
	Horse	C	++	++	+	±	−	−
		P	++	++	+	+	−	−
	Cow	C	++	++	+	±	−	−
		P	++	++	+	+	−	−
	Rabbit	C	++	++	++	+	−	−
		P	++	+	−	−	−	−
Fishes	Carp	C	±	−	−	−	−	−
		P	±	−	−	−	−	−
	Rock-fish	E. Coli	±	−	−	−	−	−
		P	±	−	−	−	−	−
	Sea-bass	E. Coli	+	±	−	−	−	−
		P	+	±	−	−	−	−

While the serums of warm-blooded animals agglutinated up to about 50–100 times the natural agglutination value, those of cold-blooded animals (fishes) agglutinated only about 12.5–25.0 times, showing that their agglutination value is lower.

III. Experimental procedure.

1.0 ml. of the original solution of each serum and serum twice diluted

with sterilized physiological natrium chloride solution are each put in a sterilized test tube. 1 platinum loopful of bacterium fluid cultured in 10.0 ml. of bouillon is successively cultured for 4 days, and then diluted with 100 ml. of sterilized physiological natrium chloride solution, and 1 platinum loopful of it is inoculated, stirred and added to each of the serums, which is put in a 32° C incubator and run into a Petri's dishes at certain times and tried mixed culture. The colony developed was calculated accurately. For control, the same quantity of serum, heated at 56° C for 30 minutes and made inactive, was used and the number of bacteria was likewise calculated and compared. The results are as shown in the Table 2.

Table 2. Bactericidal power of horse serum.

Serum	Strains	Time in hour			
Horse		0	2	5	24
Original sol.	P	216	92	36	0
Twice sol.	P	233	134	98	11
Control orig. sol.	P	230	235	346	850
„ twice „	P	218	276	378	1020
Original sol.	C	182	54	12	0
Twice sol.	C	206	128	Sapro-phyte	Sapro-phyte
Control orig. sol.	C	185	238	312	900

Table 3. Bactericidal power of cow serum.

Cow					
Original sol.	P	345	108	42	0
Twice sol.	P	300	187	140	42
Control (orig. sol.)	P	247	265	388	992
Control twice sol.	P	252	322	410	1640
Cow original sol.	C	220	90	43	0
Twice sol.	C	218	120	75	0
Control orig. sol.	C	244	258	395	1050

Table 4. Bactericidal power of carp serum.

Carp original sol.	P	240	120	84	24
Carp twice sol.	P	240	—	150	—
Control orig. sol.	P	235	240	348	744
Carp orig. sol.	C	225	162	78	8

IV. Experimental results.

The fish serum was tested up to 24 hours against both strains, and its bactericidal effect was clearly recognized. The serums of warm-blooded

animals (horse and cow in this case) are more bactericidal than those of fish. Twice-diluted serum is inferior to the original serum in bactericidal effect. In the control test in which inactive serums were used, the bactericidal effect was not recognized at all, the bacteria reproducing themselves gradually with the lapse of time.

Conclusion

The normal serum of fish is recognized to have a natural antibody and bactericidal effect. However, it is inferior in strength to the serums of warm-blooded animals (horse and cow), and though from only one instance, this has been recognized as a fact as our elders pointed out.

Owing to the difficulty of collecting much fish serum, detailed tests in relation to the functioning hours, temperature, many different kinds of bacteria and fishes could not be made. It is not proper to reach a conclusion immediately from the simple test I have made, and I must content myself with thinking that a general trend has been shown by my test.

My hearty thanks are due to Prof. T. MIMURA who helped me in collecting the fish serum, and to Assistant Prof. T. Ino of Kominato Marine Biological Station of our University who afforded me facilities. I also express my gratitude to Technical Officials. S. Ejima of the Welfare Ministry's Institute who spared me the serums various animals and strains.

LITERATURE

1. KAUFFMAN, F. 1926: Zschr. f. Hyg. Bd. 106, S. 246.
2. KIMURA, K. 1933: Marine Products Manufacturing Science (Whole collection of Science of Marine Products) Vol. 10, 81.
3. ANGRER K. 1909: Berichte ausder Kgl. Bayerischen Biologischen Versuchsstation in Munchen Bd. II, S. 143.

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ZUR ENTWICKLUNGSGESCHICHTE VON *PROCTOTREMATOIDES PISODONTOPHIDIS* YAMAGUTI, 1938.

I. MITTEILUNG. AGAMODISTOMA UND IHRE ENTWICKLUNG.

von

Toshikazu HOSHINA

(Mit 3 Tabellen 2 Abbildungen, und 1 Tafel)

(Eingegangen am 28 Okt. 1951.)

Proctotrematoides pisodontophidis YAMAGUTI, 1938 ist eine Art von den digenen Trematoden, die von Dr. YAMAGUTI (1938) im Darme des Fisches (*Pisoödontophis cancrivorous*) gefunden wurden.

Beim Aale (*Anguilla japonica*) in Urayasu, Chiba Präf., im Darme findet sich eine Art von Saugwürmer nicht selten, welche vermutlich mit *Proctotrematoides pisodontophidis* identisch ist.[1)]

Andrerseits wurden drei Arten von Agamodistomen von Dr. T. OGATA (1944,' 49)[2)]. in *Laternula kamakurana* aus dem Seeufer bei Urayasu gefunden. Eine Art von diesen Agamodistomen gleicht sehr der oben erwähnten beim Aale vorkommenden Trematodenart. Daher wird diese Art als erwachsene Form von bei *Laternula kamakurana* schmarotzender Metaceraria betrachtet.

Herr Mikio YOSHIZAWA, Student von Kaiserlischem Fischerei-Institut in höherer Kursus (gegenwärtig Tokio Universität für Fischerei) war mit dem Versuche über die Entwicklung von dieser Metacercaria unter Leitung des Dr. OGATA beschäftigt, aber, zum Unglück, am 2. Sept. 1949 ist er in dem Unfall jung gestorben. Zu ihm hatte ich enge Beziehung auf dem wissenschaftlichen Gebiete und auf Anraten des Dr. T. OGATA hatte ich auf neue eine Versuche über diesen Problem.

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1). Diese Tatsache wurde von Yoshizawa und mir in 1949 entdeckt.

2). 1944. Ogata, T.: On the morphology, ecology and life history of an agamodistome parasitic in a bivalve, *Puphia (Ruditapes) philippinarum* (ADMES ET REEVE). Sci. Rep. Tokyo Univ. Litera. Sci., Sect. B, Vol. 7., No. 102.

1949. ———: Über einiger bei der brackischen Fischen und Muscheln vorkommenden encystierten Metacercarien. Die gehaltene Vorlesung an der Generalversammlung von Japänische Parasitologische Geselschaft im April 1949.

Und durch Fütterug den Aalen mit Agamodistomen aus *Laternula kamakurana* wurden erwachsene Formen derselben Art gewonnen. Obwohl meine Forschung in der Entwicklung von den Parasiten noch in der Untersuchung steht, will ich ein Teil von Untersuchungsergebnisse hier mitteilen.

Es ist mir eine angenehme Pflicht denjenigen, die mir bei der Durchführung dieser Arbeit in irgendeiner Weise behilflich waren, an dieser Stelle meinen besonderen Dank auszusprechen. In erster Linie aber gilt mein Dank Herrn Dr. Toji OGATA. Ohne seine Anregung, sowie freundschaftliche Unterstützung, wäre die vorliegende Arbeit unmöglich gewesen. Ferner statt ich Dank Herrn Professor Densaburo INABA für seine freundliche Unterstützung beim Beschaffung des Untersuchungsmaterials und Fräulein Kinko KAMEZAKI für ihre besondere Hilfe bei der Durchführung dieser Arbeit ab. Zulezt muss ich nicht vergessen Herrn Dr. Tamao FUKUI meinen besten Dank für seinen nutzlichen Rat auszudrücken.

I. Agamodistoma.

Die Untersuchungsmateriallien (*Laternula kamakurana*) wurden bei Seeufer von Urayasu gesammelt. Häufig zusammen mit Metacercaria *Acanthopariphium* kommen die Agamodistomen in der encystierten Form in der Innenwand der Siphonalröhren von *Laternula kamakurana* vor.

Die Cysten (Tafel 1. Fig. 1.) sind Kugelförmig, liegen in den superfiziellen Muskulaturen der Siphonalröhre, und gewöhnlich halbkugelig angeschwollen.

Sie bestehen aus einer dünnen Transparenthülle (etwas 8μ dick), und ob ihre Dimensionen gleich wegen ihrer Entwicklungszustände ziemlich auch wechseln, doch sind sie mehr oder weniger 0.5 mm im Durchmesser. Infolge der Messung der Materiallien (am 20. Okt. 1949 gesammelt), in der physiologischen Kochsalzlösung mit Deckglass eingebettet, konnte ihr Durchmesser 0.340–0.617 mm bestimmt werden, und aber die Messung der gewahlten Materiallien der relativ grossen Cysten zeigte uns 0.390–0.465 mm im Durchmesser, durchschnitt. 0.430 mm, im Falle dass die Materiallien, in der physiologischen Kochsalzlösung ohne Deckglass eingebettet, gemesst wurden.

Es scheint, dass Prozentsatz der von der Agamodistomen infizierten Muscheln und der Zahl der bei einer Muschel schmarotzenden Agamodistomen ziemlich ändern sich infolge der Jahreszeit und Grösse der Muscheln.

In der Tab. 1. ist die Prozentsatz von den infizierten Muscheln nach den Untersuchungsergebnissen von dem Jahre 1950/'51 zusammengestellt.

Wie die Tab. 1. zeigt, wird es erkannt dass die Zahlen der Muscheln, die von der Agamodistomen befallen sind, vermehrten sich in Sommerzeit.

Tab. 1. Die Prozentsatz von der Agamodistomeninfektion bei *Laternula kamakurana*.*

Fangdatum des Materials	Zahl der Untersuchten Muscheln	Zahl der infizierten Muscheln	V. H.
15. Mai 1950	200	31	15.5
25. Mai 1950	100	18	18.0
26. Mai 1950	77	17	22.1
16. August 1950	95	59	62.1
17. Sept. 1950	40	36	90.0
14. Okt. 1950	31	31	100.0
22. April 1951	56	33	59.1
5. August 1951	95	59	62.1
14. August 1951	32	25	78.1
31. August 1951	37	37	100.0

* Die Grösse der untersuchten Muscheln war 26–51 mm lang (durchschnitt. 40.7 mm)

Im allgemeinen, in Hinsicht auf die Beziehung zwischen Grösse der Muscheln und ihrer Agamodistomeninfektion ist bei grossen Muscheln der Infektionsprozentsatzes hoch und auch die Zahl der Agamodistoma im Individuum viel. In der Regel finden sich die Agamodistomen in einer Muschel wenig, aber man findet selten bei ihren starken Invasionen, mehr als hundert.

Im Maximum wurden 116 Agamodistomen in einer Muschel unter den am 14. Okt. 1950 gesammelten Materiallien beobachtet. Wie bereits erwähnt, kommt die Agamodistoma bei der Innenwand der Siphonalröhre von Wirtstieres vor, aber bemerkenswert ist es, dass sie sich besonders häufig beim Kloakalsipho findet. In der Tab. 2. weist die parasitischen Verhältnisse im Falle von den 183 untersuchten Muscheln auf. Diese Materiallien sind am. 16. August 1950, 22. April, 1951, 14. August und 31. August 1951 gesammelt.

Tab. 2. Parasitische Verhältnisse von Agamodistomen in *Laternula kamakurana*.

Zahl der untersuchten Muscheln	Zahl der infizierten Muscheln	Zahl der Muscheln nur bei Branchialsipho infiziert	Zahl der Muscheln nur bei Kloakalsipho infiziert	Zahl der Muscheln bei beiden Siphonen infiziert
183	117	5	95	17

Und die Gesmatzahl der Agamodistomen war 31 beim Branchialsipho und 676 beim Kloakalsipho.

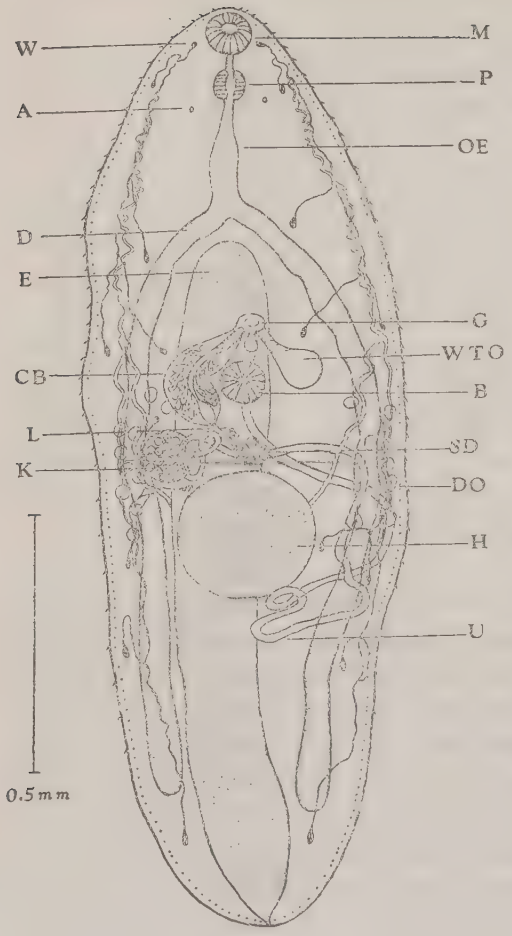


Abb. 1. Die Larve aus *Laternula kamakurana*. Ventralansicht.

In der Cyste liegen die Agamodistomen mit ihrer Längsachse nach Bauchseite hinbiegend.

Sie können leicht aus der Cyste herausgezogen werden und langsame Bewegung der heraus gezogenen Larve wird beobachtet. In dem körperlichem Bau ist die vollentwickelten Larve wohl differenziert und hat keine merkwürdige Verschiedenheiten als erwachsener Wurm. Sie ist der *Proctotrematoides pisodontophidis* YAMAGUTI sehr ähnlich und ich beobachte es wahrscheinlich als ihre Larve, denn die Cuticula hat die feinen Stacheln in transversale Reihen, in der Nähe des Pharynx findet sich ein Paar Augenpunkte, und ferner in der Form und Stellung des Verdauungssystems, des Bauchsaugnapfs, der Geschlechtsorgane, deren beitragenden Organe, und der Excretionsblase etc. stimmen diese morphologische wichtige Charaktere mit dieser Art überein.

Folgend darlege ich Punkt für Punkt über die Charakteristik der Larve (Abb. 1).

Im frischen Zustande, Körper 0.70–1.60 mm lang und 0.31–0.60 mm breit. Haut, Hinterende ausgenommen, mit in transversale Reihen angeordneten feinen Stacheln, besitzt, im Vorderkörper zahlreiche Keulenförmige Hautzellen. Mundsaugnapf (M), am anterioren Ende des Körpers stehend, 0.075–0.128 mm × 0.090–0.143 mm gross; Mundöffnung in der Bauchseite. Prepharynx 0.024–0.04 mm lang; Pharynx (P) 0.047–0.075 mm × 0.043–0.070 mm gross; ein Paar Augenpunkte (A) nahe beiden Seiten des Pharynx. Oesophagus (OE) 0.11–0.17 mm lang und verwandelt sich im Darmschenkel (D) beinahe in der Mitte zwischen Mundsaugnapf und Bauchsaugnapf gegabelt. Seine Hinterende erreicht bis zur Nähe des Hinterendes des Körpers. Bauchsaugnapf (B) 0.078–0.117 mm im Durchmesser und im drei Fünftel des Körpers in der Höhe.

Hoden (H) rundlich und einzig, 0.140–0.267 mm × 0.160–0.322 mm gross, Median, am Hiterende in mitterem Drittel von Körperlänge, aber bei junger Larve liegt er mehr hinterer (s. Tafel 1. Fig. 2.). Er entsendet zwei Vas deferens von dessen Vorderrande aus. Cirrusbeutel (CB) biegend und auf der rechten Seite des Bauchsaugnapfs; ihr Grösse 0.190–0.335 mm in der grossen Achse und 0.070–0.093 mm dick. Cirrus mit langen Haaren; prostatische Zellen wohl entwikelt; Vesicula seminalis 0.055–0.07 mm × 0.035–0.049 mm gross. Keimstock (K) rundlich und rechtseitig vor dem Hoden, sein Grösse 0.070–0.128 mm × 0.075–0.117 mm. Schalendrüsenkomplex (SD) auf der linken-dorsalen Seite des Keimstocks zwar auf der dorsale Seite vor dem Hoden.

LAURER'scher Kanal (L) öffnet sich auf Mediane-Dorsalseite hinter dem Cirrusbeutel. Dotterstöcke (DO) auf beiden Seiten des Körpers in der Strecke von Bauchsaugnapf zum Hoden verteilend; quere Dottergänge mit einander verbindet und dicht vor dem Hoden gelagert. Uterus (U) windet sich auf der linken Seite des Körpers durch die Strecke bis aus Hinterende des Hodens oder reicht etwas hinterer; Endstück des Uterus steigt entlang der Körpermitte auf und endlich verbindet sich mit dem weiblichen Terminalorgane. Weibliches Terminalorgan (WTO) linke Seite von Bauchsaugnapf, 0.119–0.220 mm × 0.083–0.110 mm gross, ihr proximaler Teil lang haarig an innerern Seite.

Genitalporus (G) mit dickem Haut umgeben und fast auf der linken Seite vor dem Bauchsaugnapf. Ein flacheförmiger Sack zwischen Cirrusbeutel und weiblichem Terminalorgan verbindet sich mit Genitalsinus. Dessen äussren Durchmesser 40 μ mehr oder minder, und an Innenseiten mit langen Haaren versehen. Exkretionsblase (E) lang und breit, undurchsichtig wegen der mit Körner gefüllten Substanzen, nahe an der Dorsalseite im Mittelfelde des Körpers, ihr Vorderende an die Darmgabelung reichend; Exkretionsporus an dem Hinterende des Körpers.

Röhrensystem mag vielleicht wie folgend: die Exkretionsblase entsendet ein Paar Hauptsammelröhren nach vorn aus etwas hintererer Höhe als die Hälfte des Körpers, die bis in die Höhe des Bauchsaugnapfs, order etwas höher aufsteigen und mit denjenigen verbunden sind, die entlang der beiden Seiten des Körpers laufen Sammelrörchen, und im Laufe ihrer Gänge teilen sich in zwei Äste ihre oberen und unteren Teile von dieser Verbindungspunkte und beziehungsweise tragen die in den beiden Enden des Körpers verteilten Äste zwei Terminalzellen und die in der Mitte der beiden Seiten verteilten Äste drei Terminalzellen. Darum wird die Formel der Wimperzellen $[(2+3) + (3+2)] \times 2$ dargestellt.

2. Entwicklungsversuche der Agamodistoma bei Aalen.

Die Versuche wurden von Mai bis zum Juni 1950, Okt./Nov. 1950 und April/Juni 1951 angestellt.

Die zu den Versuchen benutzten Aale wurden in Yoshida Fischzucht Anstalt in Sizuoka Präf. gezücht, der Tokio Universität für Fischerei gehört. Die zu den von Mai bis zum Juni 1950 angestellten Versuchen gebrauchteten Aale kamen zur Hand in Sept. 1949 und bis zum Versuchsbeginn waren sie in dem Hungerzustande gehalten. Die zu den von Nov. 1950 Juni 1951 angestellten Versuchen benutzten Aale kamen am 25. Sept. 1950 zur Hand und sie waren bis zum Versuchebeginn in dem gleichen Zustande gehalten. Ihr Körpergewicht war 40~95 gr. und Körperlänge 35~46.5 cm.

Ins kleine Aquarium gesetzten Aale essen gar nicht das gegebene Futter, und um den Versuchsfisch die Agamodistomen essen zu lassen, werden sie mit der 0.2% Urethan narkotisiert und in ihre Magen wurden einige infizierte Siphonalfleischestücke mit Agamodistomen von *Laternula kamahurana*, mit der Pinzette eingefügt. Nachdem wurden diejenigen Aale ins Glassaquarium je ein Fisch eingeliefert, welche sie sich nicht innerhalb 48 Std. erbrochen hatten, und dann wurden sie im Süsswasser gehalten. Auch, bei der Nahrungszufuhr für sie wurde dieselbe Methode angewendet. Sie wurden einmal für einige Tage gefüttert. Das benutzte Futter war die Fischfleische und die Muschelfleische. Wenn die Aale Futter frassen, so wurde in der Regel das Ausscheiden des Kotes bis am Tage darauf beobachtet.

Die Untersuchungsergebnisse sind in der Tab. 3. zusammengestellt.

Tab. 3. Entwicklung der bei *Anguilla japonica* experimentalisch geschmarotzte Agamodistoma, die *Laternula kamakurana* parasitisch ist.

Vers. Nr.	Zeit des Versuchs-beginns	Zeit der Sektion	Wasser-temp. °C	Befund bei der Sektion	Bemerkungen
1.	16. Mai 1950	22. Mai 1950	20–24 durch-schit.. 21.5	Im Darme, wurden fünf Würmer gefunden sie alle unreif, aber deren Exkretion-sblase transparent waren und die Uterus etwas mehr entwickelte sich als encysti-erte larve.	Der Fisch mit acht zysten von Agamodisto-men gefüttert.
2.	26: Mai 1950	2. Juni 1950	20–22 〃 21.7	Im Darme, sechs Würmer gefunden ; ihr Entwicklun-gszustand beim übergeschri-ebenen ähnlich.	Der Fisch mit zehn Zysten von Agamodi-stomen gefüt-tert

3.	26. Mai 1950	4. Juni 1950	20–24 " 21.8	Sechs unreife Würmer wurden aus dem Darme gefunden. Ihr Exkretionsblase wurde transparent. Zahlreiche Samenfaden wurden in den weiblichen Terminalorganen einiger Würmer beobachtet, aber ihere Uterus gehalten keine Keimzellen.	Der Fisch mit dreizehn Zysten von Agamodistomen gefüttert.
4.	26. Mai 1950	10. Juni 1950	19–24 " 21.8	Acht reife Würmer wurden aus dem Darme gewonnen, aber ihre Eiererzeugungen waren nicht genug. In ihrem Uterus wurden etwa hundert Eier enthalten.	Der Fisch mit zwölf Zysten von Agamodistomen gefüttert.
5.	16. Mai 1950	15. Juni 1950	20–24 " 21.6	Am 12. Juni wurden die Eier der gefütterten Distomen zuerst im Kote des Fisches gefunden (28 Tagen nach Versuchsbeginn). (s. Tafel 1. Fig. 3.). Bei der Sektion, wurden drei vollentwickelte Würmer aus dem Darme bekommt.	Der Fisch mit neun Zysten von Agamodistomen gefüttert.
6.	14. Okt. 1950	16. Okt. 1950	19–22 " 20.6	Am 16. Okt. wurde der Fisch aus dem Zufalle tot gefunden. Bei der Sektion, wurden vier unreife Würmer aus dem Darme bekommt. Sie alle zeigten keine Fortschritten in der Entwicklung.	Der Fisch mit zehn isolierten Zysten von Agamodistomen gefüttert.
7.	"	24. Nov. 1950	10–22 " 15.3	Drei unreife Würmer wurden aus dem Darme bekommt, ihr Exkretionsblase transparent, etwas mehr entwickelte Uterus gehalten Keimzellen. (42 Tage nach Versuchsbeginn)	"
8.	23. April 1951	8. Juni 1951	10–23 " 17.0	Am 6. Juni wurden die Eier der Distomen im Kote des Versuchsfisches erwiesen (43 Tage nach Versuchsbeginn) Bei der Sektion wurden zwei vollentwickelte Würmer im Darme gefunden.	Der Fisch mit sechs Zysten von Agamodistomen gefüttert.
9.	"	10. Juni 1951	10–23 17.1	Am 8. Juni, wurden die Eier der Distomen im Kote des Versuchsfisches gefunden. (45 Tage nach Versuchsbeginn) Bei der Sektion wurden vier vollentwickelte Würmer bekommt.	Der Fisch mit elf Zysten von Agamodistomen gefüttert.

Aus der Tab. 3. geht es hervor, dass durch Fütterung der Aale mit der Agamodistomen die sämtlichen untersuchten Aale von denselben Distomen befallen wurden. Und durchschnittlich wurden 46% von den gefütterten Agamodistomen im Darme des Aales wiedergefunden. Ferner könne ich feststellen, dass sie sich im Körper des Aales wohl entwickeln. Daher werden die Aale als ihre geeignete Endwirte betrachtet. Die durch Aale eingenommenen encystierten Agamodistomen verlassen die Zysten und sie wandern vou Magen zum Darme ein. Denn Versuche Nr. 6 nach wurden die aus der Zysten herausgetretenen Agamodistomen am zweiten Tage nach dem Versuchsbeginn im Darme gefunden. Der Entwicklungsverlauf der Agamodistomen ist verhältmässig langsam. Bei einer Temperatur von 19-24° C (durchschnitt. 22° C) in etwa neun Tagen kam der männliche Geschlechtsorgan zur reife und binnen zwei Wochen beginnte die Formation der Eier und in vier Wochen wurden die Agamodistomen vollentwickelte Würmer und dann machten sie den Anfang mit dem Eierlegen, seitdem sie in ihre Endwirte hineingetreten. Es scheint, dass Entwicklung der Agamodistomen eng mit der Wassertemperatur sowie dem Ernährungszustande des Wirts verwandt ist; z. b., bei einer Temperatur von 11-22° C (durchschnitt. 17° C) bis zum Beginn des Eierlegens haben sie 42 Tage im Falle des Versuchs 8. und 43 Tage im Falle des Versuchs 9. beziehungsweise, gebraucht.

Wenn es auch über 40 Tage nach Versuchsbeginn Vergangen wäre, dennoch sind die Distomen beim Versuche 7.* im Larvenstadium geblieben. Vermutlich wurden diese Tatsache, glaube ich nicht nur durch niedrige Temperatur sondern auch durch den Hungerzustand des Wirts bedingt. Nämlich ist es wahrscheinlich, dass die Agamodistomen bei ihrem Wirte im Hungerzustande nicht entwicklen können.

Diese Agamodistomen sind im Salzwasser vorkommende Art, aber sie können jedoch beim Wirte im Süsswasser entwickeln, wenn sie die Gelegenheit finden in ihre Endwirten hineinzuwandern. Durch Entwicklung sind die Agamodistomen nicht so sehr veränderlich. Die hauptsächlichen Veränderungen sind folgend: die durchsichtige Verwandlung der Exkretionsblase, die merkwürdige Entwicklung der Uterus und Entwicklung der Dotterstöcke etc. Die Grösse des Körpers verändert sich nicht so sehr.

* Wegen ihrer vomitio konnte man nicht die Aale in dem Versuche 6. und 7. Agamodistomen samt ihrem Wirtsfleische füttern, daher war ich gezwungen, in ihre Magen die aus ihrer Wirtsgewebe isolierten Agamodistomenzysten samt physiologischer Kochsalzlösung mittels der Spritze einzuspritzen. Da es der Fall war, war es unmöglich auch während des Versuchszeitraumes dem Aale von Versuche 7. Futter zu geben.

3. Ausgebildeter Wurm.

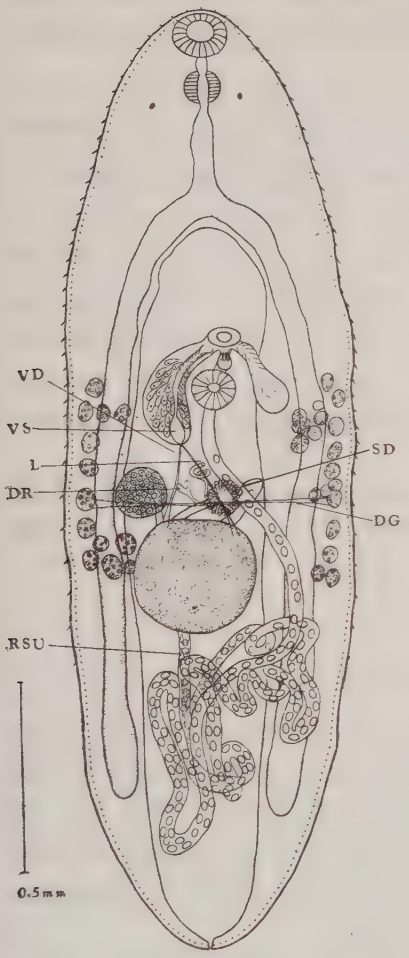


Abb. 2. Der aus dem Darme des Aals gefundene Wurm, der durch Fütterung dem Fischen mit der Agamodistomencysten gewonnen wurde. (Aus Versuche 4.; 15 Tage nach Versuchsbeginn). Ventralansicht.

DR Dotterreservoir
DG Querer Dottergang
L LAURER'scher Kanal
RSU Receptaculum seminis uterinum
SD Schalendrüsen
VD Vas deferens
VS Vesicula seminalis

Der erwachsene Wurm (Abb. 2 und Tafel 1. Fig. 4.) ist folgend: Im frischen Zustande ist sein Körper 1.37~2.44 mm lang und 0.59—0 71 mm breit. Die Körperoberfläche ist, Hinterende aus genommen, mit feinen Stacheln in transversalen Reihen besitzt. Zahlreiche keulenförmige Hautzellen finden sich im Vorderköper wie bei der Larve (s. Tafel 1. Fig. 4). Die am Ende stehende Mundsaugnapt ist 0.107—0.160 mm lang und 0.139—0 171 mm breit: der Prepharynx 0.032—0.040 mm lang; der Pharynx 0.064—0.075 mm× 0.053 - 0·073mm gross. Ein Paar Augenpunkte finden sich nahe bei dem Pharynx. Der Oesophagus ist 0.035—0224 mm lang und er gabelt sich beinahe in der Mitte zwischen Mundsaugnapf und Bauchsaugnapf; die Darmschenkel erreicht zum Hinterende des Körpers. Der in der Mediane und im drei Fünftel in der Höhe des Körpers liegende Bauchsaugnapf ist 0.107—0.133 mm im transversalen Durchmesser. Der Hoden ist rundlich, 0.149-0.267 mm × 0.267—0.330 mm gross, ihr Besassplatz ist wie bei der Larve. Der Cirrusbeutel ist 0.400-0.440 mm in der grossen Achse und 0.110—0.140 mm dick. Das Vesicula seminalis (VS) ist 0.055-0.060 mm lang und 0.033-0.049 mm breit. Der Besassplatz des Genitalporus ist wie bei der Larve und von dickem Haut umgeben und öffnet sich vor dem Bauchsaugnapf etwas linkwärts von der Medianlinie. Das zwischen Cirrusbeutel und weiblichem Terminalorgan liegende flaschförmige Sack ist im äussieren Durchmesser 40-50 μ. Der rundliche Keimstock ist 0.149-0.171 mm×0.107-0.160 mm

gross und liegt auf rechter Seite des Körpers kurz vor dem Hoden. Der Besassplatz des Schalendrüsenkomplex ist wie bei der Larve. Der Uterus besitzt den Raum zwischen dem Hinterranden des Hodens und dem Hinterende des Körpers und ihr Endeteil läuft durch linke Seite des Hodens nach vorn bis zur Mediane, dann geht endlich ins Metraterm über. Das Receptaculum seminis uterinis (RSU) ist vorhand. Das weibliche Terminalorgan ist 0.119–0.220 mm lang und 0.100–0.129 mm breit. Die Dotterstöcke bestehen aus vielen acini und sind auf beiden Seiten des Körpers in der Strecke zwischen von Bauchsaugnapf bis zum Hoden verteilt. Die Zahl der im Uterus enthaltenen Eier ist eine grosse Anzahl. Sie sind elliptisch, ihre Farbe ist gelblich. Im frischen Zustande haben die Eier eine Länge von 23.9–30.3μ und eine Breite von 13.3–16.5μ (Taf. 1. Fig. 3.) Die Exkretionsblase ist 1.18–1.58 mm× 0.064–0.068 mm gross.

Die oben erwähnten Charaktere der Art stimmen whol mit den Charaktere der *Proct. pisodontophidis* YAMAGUTI, 1938 überein. Deshalb schläge ich die Identifizierung der vorstehenden Art mit *Proct. pisodontophidis* YAMAGUTI, 1938 vor.

4. Zussammenfassung.

Durch Fütterug der Aale mit einer Art von der bei aus Urayasu Gegend stammenden *Laternula kamakurana* schmarozenden Agamodistomen werden dieselben erwachsenen Formen gewonnen. Der erwachsene Wurm wird mit *Proct. pisodontopidis* YAMAGUTI, 1938 identifiziert. Die Agamodistomen kommen bei der Innenwand des Sipho von *Laternula kamakurana* vor, hauptsächlisch beim Kloakalsipho, in der encystierter Form.

Die Cysten sind kugelförmig und von dünner Transparenthülle umgeben. Im frischen Zustande haben die vollentwickelten Cysten Durchmesser von etws 0.5 mm.

Es scheint, dass der Infektionsprozentsatz von der Agamodistomen bei *Laternula kamakurana* sich in Sommerzeit vermehrt.

Die Entwicklungsverlauf der Agamodistomen beim Aale wurde beobachtet wie folgt: bei einer Temperatur von 19–24° C (durchschnitt. 22° C), in einer Woche wurde die Exkretionsblase durchsichtig, in etwas neun Tage kam zur Maturiät das männliche Geschlechtsorgan, das weibliche Organ wurde reif binnen zwei Wochen und in vier Wochen machte er den Anfang mit dem Eierlegen. Aber bei einer Temperatur 10～23° C (durchschnitt. 17°C) bis zum Beginn des Eierlegens gebrauchten sie 43～45 Tage.

Bei Urayasu Gegend, in natürlichem Zustande werden die Aale als ein Endwirt von diesen Agamodistomen betrachtet.

LITERATURVERZEICHNIS.

1. YAMAGUTI, S.: Studies on the Helminth Fauna of Japan. Part 2. Trematodes of Fishes, I. Jap. Jour. Zool., Vol. 5, 1934.

2. ——————: Studies on the Helminth Fauna of Japan. Part 21. Trematodes of Fishes, IV. Published by authar. March 10, 1938. in Kyoto, Japan.

TAFELERKLÄRUNG.

Tafel 1.

Fig. 1. Die Cyste der Agamodistoma von *Proctotrematoides pisodontophidis* YAMAGUTI, 1938, aus *Laternula kamakurana.*

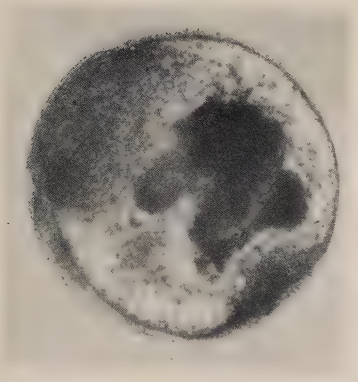
Fig. 2. Die aus der Cyste herausgezogene Larve von *Proct. pisodontophidis*; etwas jung. Dorsalansicht.

Fig. 3. Die Eier derselben Art, die im Kote des Aals beim Versuche 5. gefunden wurde. (28 Tage nach Versuchsbeginn).

Fig. 4. Der beim Aale vollentwickelte Wurm von *Proct. pisodontophidis*, der durch die Aalen Fütterung mit der Agamodistomen aus *Laternula kamakurana* gewonnen wurde (30 Tage nach Versuchsbeginn, beim Versuche 5.). Dorsalansicht.

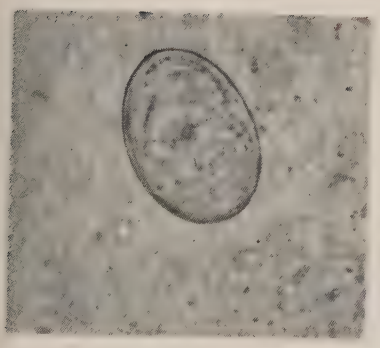
PL. I.

1



0.3 mm

3

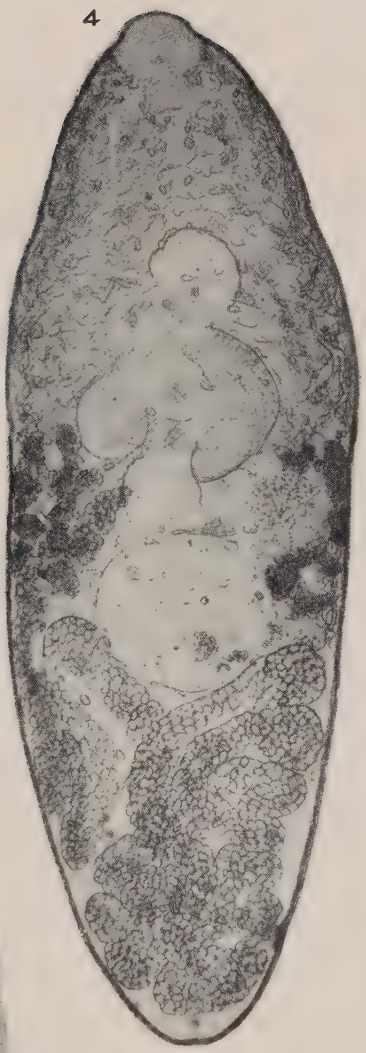


0.02 mm

2

0.2 mm

4



0.5 mm

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SOME MACRUROUS DECAPOD CRUSTACEA FOUND IN JAPANESE WATERS, WITH DESCRIPTIONS OF FOUR NEW SPECIES

Itsuo KUBO

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Included in the present report are descriptions and discussions of the following eight species basing on specimens collected from various localities by many people, viz., *Parapenaeus longipes* ALCOCK, *Solenocera utinomii*, sp. nov., *Athanas setoensis*, sp. nov., *Periclimenes* (*Ancylocaris*) *tosaensis*, sp. nov., *Ogyrides striaticauda* KEMP, *Spirontocaris crassirostris* sp. nov., *Lysmata* (*Hippolysmata*) *kükenthali* (DE MAN *Lysmata* (*Hippolysmata*) *vittata* (STIMPSON). Of these four species, ones are new to science, three ones, except *Lysmata* (*Hippolysmata*) *vittata*, are known species as new Japanese records.

Moreover, a noticeable matter found in this paper is that the genus *Hippolysmata* (STIMPSON, 1860) has been combined to the genus, *Lysmata* (RISSO, I826) for the reason given later on.

It should be given herewith that the body-length used in this paper is represented by the distance measured from the post-orbital margin of carapace to the tip of telson. The type specimens of the two new species, *Solenocera utinomii* and *Athanas setoensis* have been deposited in the collections of the Museum of Seto Marine Biological Station of Kyoto University, and the other two ones of *Periclimenes* (*Ancylocaris*) *tosaensis* and *Spirontocaris crassirostris* have been deposited in the collections of the Fisherybiology laboratory of the Tokyo University of Fisheries.

This opportunity is taken to the people by whom the specimens dealt with were collected and placed at my disposal. I must also to express my thanks to the Ministry of Education by which the present investigation has undergone the financial aid.

Parapenaeus longipes ALCOCK

(Figs. 1–3, Tab. 1)

Parapenaeus longipes, ALCOCK, 1905, p. 525; 1906, p. 33–34, pl. 6, figs. 18, 18a–b.

The specimens at the disposal of the present author are discrepant with the figure (Pl. 6, fig. 18) given by Alcock for his *Parapenaeus longipes* in only a point being as follows. As to the antennular peduncle, the second segment is about twice as long as the third one in the present specimens. Whilst it is shown to be about one and half times the length of the third segment in the

illustration (description of this matter is devoid) given by ALCOCK. The present specimens are, however, well in agreement with the description and figures made by ALCOCK in all other important charactristics. Therefore the specimens examined are tentatively referred to the present species.

The description and figures worked out by ALCOCK are excellent and sure, but those of some characteristics are less sufficient. Moreover some traits important for the taxonomy of this species are overlooked by him. So that in the following lines some additions for the description may be done on the color pattern of pleon, number of rostral spines, length of antennular flagella, mouth-parts, sexual organs of both sexes, appendix masculina, and stomodaeal ossicles.

The first 4 abdominal segments are furnished with narrow cross bands of rather light reddish brown color near posterior border of lateral surface of each segment in alcoholic specimens (Fig. 1).

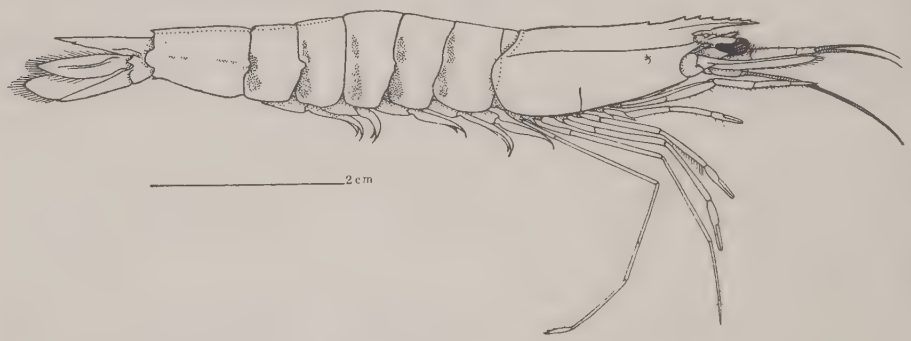

Fig. 1. *Parapenaeus longipes* ALCOCK, male.

Pleonal segments from fourth to sixth are sharply carinated along dorsomedian line, and the carina of each segment ends posteriorly into minute acute spine.

able 1. Measurements in mm and counts of bodily parts of *Parapenaeus longipes* ALCOCK. F, female; M, male; *, inserted on carapace.

No.	Sex	Body-length	Carapace length	No. of rostral spines	Length of antennular flagella Upper	Lower
1	F	72.0	20.0	6+1*	9.3	12.2
2	„	——	18.5	7+1	9.5	13.5
3	„	66.0	18.0	6+1	10.0	12.4
4	„	65.5	17.5	6+1	9.0	12.0
5	„	63.0	17.3	6+1	8.5	12.4
6	„	——	16.4	6+1	8.2	11.2
7	„	60.0	16.0	6+1	7.3	10.0
8	„	——	14.0	6+1	6.5	8.0
9	M	56.8	15.0	6+1	14.5	17.4
10	„	——	14.8	6+1	12.0	14.0
11	„	44.0	10.8	6+1	4.5	——

Rostral spines including one inserted on carapace vary from 7 to 8 in number, but usually 7, in 1 out of 11 specimens dealt with has 8 ones (Table 1).

Upper antennular flagella 20–25% shorter than lower ones in both sexes. The flagella show remarkable sexual dimorphism in length as set forth already by ALCOCK, much longer in male as shown in Fig. 2. The difference seems to appear already at the time when the carapacial length measures about 11 mm (about 44 mm in body-length).

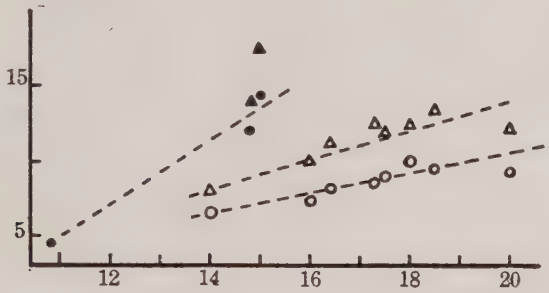

Fig. 2. Lengths of antennular flagella (ordinate) of *Parapenaeus longipes* in relation to the carapecial length. Circles, upper flagella; triangles, lower ones; solid marks, male; soft ones, female.

Antennular flagella 1.4–1.5 times as long as body-length in specimens measuring 60–70 mm in body-length.

Mandible well developed, composed of well calcified molar and incisor processes and an endopodite. Incisor process provided with two illdefined teeth bearing bluntly pointed apex on cutting edge, and armed with a similar tooth on each anterior and posterior angle. Distal segment of endopodite about 1.6 times as long as wide (Fig. 3, *A–C*). Maxillulary palp (Fig. 3, *D*) rounded on distal end like that of *Parapenaeus fissurus* (KUBO, 1949, fig. 143, *A*), but the tubercle found on the basal outer border is less developed. Maxillae have anterior and posterior proximal endites, anterior one distally divided into two lobules, but the other one not divided (Fig. 3, *E*); endopodite not jointed, has a finger-like process on disto-median edge. Second maxilliped J-shaped, 7-jointed; merus about half the length of this maxilliped, ca. 2.6 times as long as wide. Third maxilliped pediform, reaches to distal rim of antennal scale in a female measuring 72 mm long, 7-segmented; ischium 0 8, merus 0.6, carpus 0.3, propodus 0.28, dactylus 0.51 in proportion to basis in length, ischium about 4.8 times as long as wide in the same specimen given above.

Petasma about 3.8 times as long as wide, furnished with petaloid apical lobules. Of those apical lobules, the lateral ones are twisted forming a horn-like process directed forwards with a lumen, have a pointed short process directed distomedially on distal dorsal edge. Median lobes have a pair of thumb-like protuberance directed backwards on dorsal lateral border near

proximal end. Lateral lobes with a pair of prominent pointed spines directed strongly inwards on dorso-lateral surface just behind the base of the horn-like apical process (Fig. 3, *F*, *G*).

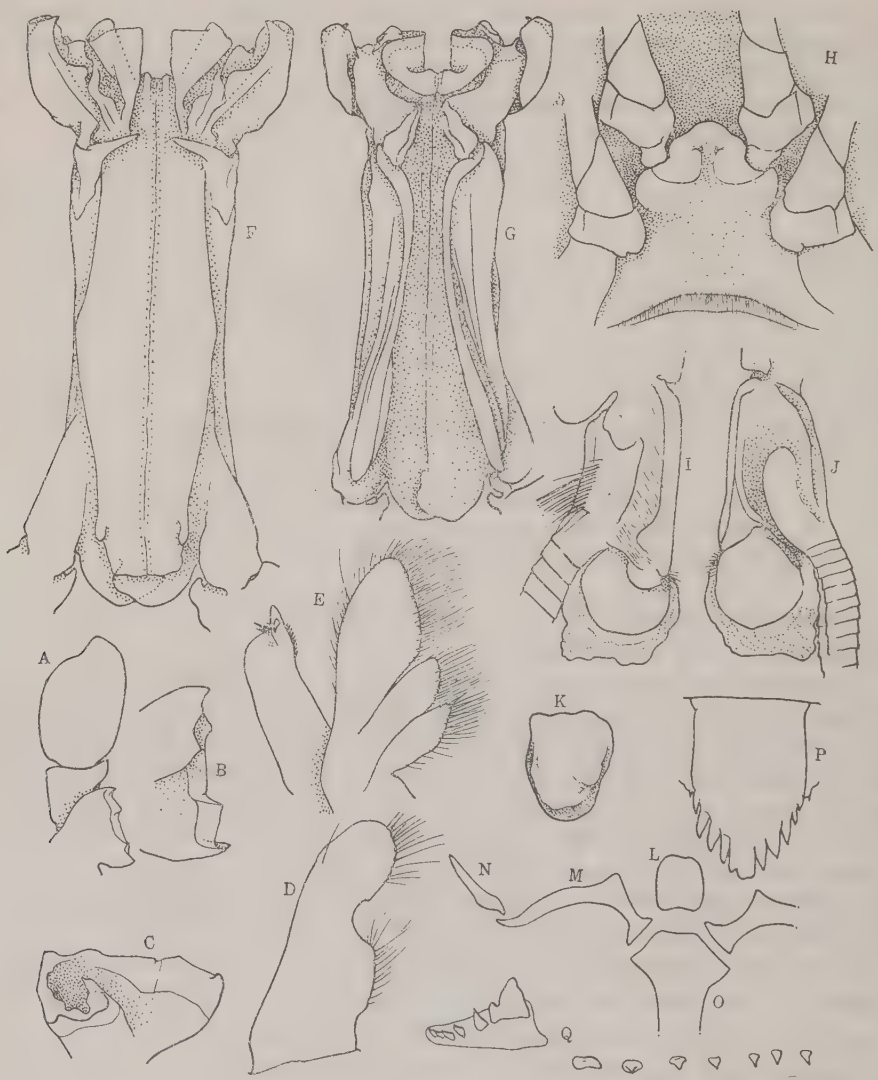

Fig. 3. Bodily parts of *Parapenaeus longipes*. *A*, ventral aspect of mandible, × 6; *B*, dorsal view of mandible without palp, × 10; *C*, inner view of mandible without palp, × 10; *D*, endopodite of maxillula, × 17.5; *E*, endopodite and inner laciniae of maxilla, × 17.5; *F*, dorsal aspect of petasma of a male, 57.0 mm in body-length, × 14; *G*, ventral aspect of petasma of the same male, × 11.5; *H*, thelycum of a female, 66 mm long, × 6; *I*, anterior view of appendix masculina of a male, 57 mm long, × 14; *J*, posterior view of the same appendage of fig. *I*; *K*, apical surface of distal piece of appendix masculina, × 14; *L*—*O*, cardiac, pterocardiac, prezygocardiac, and anterior part of urocardiac ossicles respectively, × 17.5; *P*, Urocardiac-prepyloric ossicle, × 25; *Q*, some teeth found on posterior part of cardiac plate and zygocardiac ossicle, × 25.

Thelycum nearly resembles that of *Parapenaeus investigatoris* in general feature having anterior and posterior parts. Anterior part shows a trilobate condition on anterior border, somewhat medially concave. Posterior part flattish on ventral surface (Fig. 3, *H*).

Appendix masculina similar to that of other species of the genus *Parapenaeus*; distal piece bears a transparent membranous flap along lateral and distal margins, apical surface flattish and rather strongly lifted (Fig. 3. *I-K*).

Stomodaeal ossicles show little deviation in their configuration comparing with those of other species which belong to the genus *Parapenaeus*. Lateral plate of cardiac chamber of stomach armed with 22-23 pointed teeth arranged in a straight longitudinal line. Zygocardiac ossicle with 6-7 sharply pointed teeth decreasing in size posteriorly (Fig. 3, *L-O*). Urocardiac-prepyloric ossicle provided with 11–13 teeth along distal margin (Fig. 3, *P*).

The description given above is based on 8 females and 3 males taken by Mr. J. Yasuda during the period from Oct. 25 to Dec. 26, 1950. Females range from ca. 55 to 72 mm in body-length (14–20 mm in carapecial-length), males measure 44–56.8 mm in dody-length (10.8–15.0 mm in carapace-length).

Locality; Kii Channel, at a depth of 60 m; bottom, mud.

Distribution; Indian Ocean [Off Ganjan coast, 7–35 fathoms; Malabar coast, 26–31 fathoms; Sandhead (Hooghly River)] (Alcock).

Remarks: The following matters are worthy of mentioning in this place. Firstly the posterior proximal endite of the maxilla of the present species is simple instead of bearing two apical lobules. It that the endite in question has two apical ones is found to be common to all the examined species of the genus *Parapenaeus*, viz., *P. fissurus* (Bate), *P. lanceolatus* Kubo, *P. sextuberculatus* Kubo, *P. investigatris* Alcock and Anderson (Kubo, 1949, p. 35). Secondly the spine (Spine-e in the paper of Kubo, 1949, pp. 69–70) attached to the dorsal lateral border of the median lobe of the petasma is directed backwards. This matter also appears to be peculiar to this species so far as my investigation goes. At least in *P. fissurus*, *P. sextuberculatus*, *P. lanceolatus* and *P. investigatris*, the spine is directed laterally or somewhat postero-laterally.

Finally it may be said that the present species stands much far from common forms of the genus, to which the present species is embraced, in having such peculiarities mentioned above in addition to having no spine on the antero-lateral angle of the carapace.

Solenocera utinomii sp. nov.

(Fig. 4)

Body glabrous. Carapace rounded on antero-lateral angle, armed with epigastric, supraorbital, post-orbital, antennal, and hepatic spines. Supra-

orbital spine minute, developed on post-orbital margin. Antennal one rather minute. Epigastric spine stands at one-fourth of carapace. Cervical groove well developed. Hepatic carina defined in short distance, its anterior portion runs obliquely downwards reaching to antero-lateral angle of carapace. Rostrum straight, a fourth the length of carapace, about three times as long as wide, pointed at tip; upper border straight, but lower one recurves upwards. Rostral spines 5 in number, posterior 2 of which are inserted on carapace (Fig. 4, *A*).

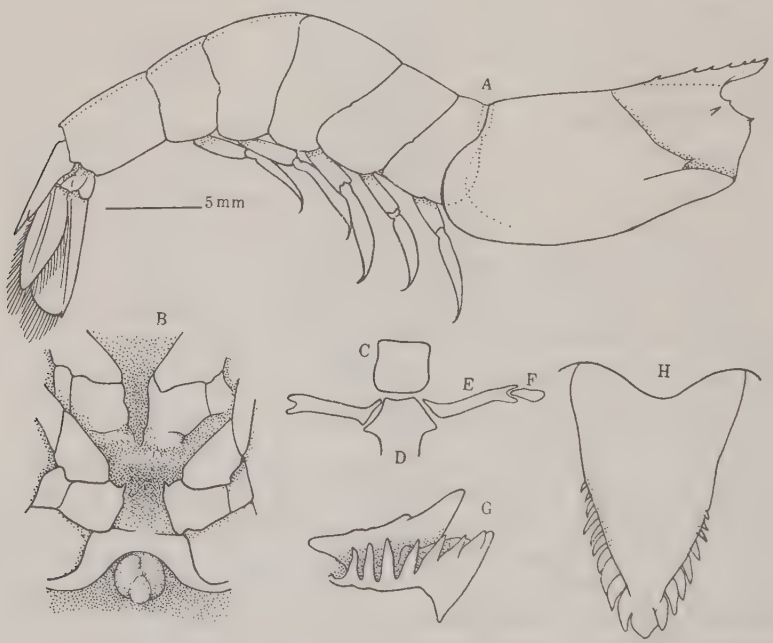

Fig. 4. *Solenocera winomii*, sp. nov. *A*, lateral aspect of body of the type specimen, female; *B*, thelycum, × 7.5; *C—F*, cardiac, anterior part of uro-cardiac, ptero-cardiac, and prezygo-cardiac ossicles respectively, × 15; *G*, zygo-cardiac ossicle, × 40; *H*, prepyloric ossicle, × 40.

Abdomen ca. 2.3 times as long as carapace, has a well defined dorso-median carina on each segment from fourth to sixth, the carina of sixth segment ends in a minute pointed spine; sixth segment a little shorter than half of carapace, about 1.4 times as long as wide. Telson somewhat shorter than half the length of carapace.

Mouth-parts show no characteristics compared with those of other species dealt with in the paper worked out by the present writer (1949).

First pereiopod 13.5 mm in length, extending a little beyond tip of rostrum. Fourth one measures 25 mm in length. Fifth one has a small pointed spine on inner edge of coxa (Fig. 4, *B*).

Thelycum not developed on thoracic sternite between fourth and fifth pereiopods, but a circular elevation is found to exist on first pleonic sternite, upper surface of the elevation found to be divided into three regions as seen in Fig. 4, *B*.

Stomodaeal ossicles resemble those of other species of the genus to which the present species belongs (Figs. 4, *C–H*). Zygocardiac ossicle with eight large teeth on inner row. Urocardiac-prepyloric one armed with 8–10 teeth on each lateral margin.

Type specimen: badly mutilated female, without eyes, antennae, antennules, third maxilliped, legs except first and fourth ones; secured in March, 1944, by Dr. H. Utinomi, to whom the scientific name of this speceis is dedicated, 39 mm in body-length, 12 mm in carapecial length.

Type locality: off Kii Peninsula, at a depth of ca. 150 m.

The present form is very similar to *Solenocera brevipes* Kubo (1949), especially in the form of rostrum, hepatic carina, and antero-lateral angle of carapace. But it greatly differs from *S. brevipes* in the shape of thelycum and third pleonic somite. The pleonic somite in question is not provided with dorso-median carina in the present species, but furnished with that in *S. brevipes*.

Athanas setoensis sp. nov.

(Figs. 5, 6)

Rostrum straight, with sharply pointed tip, extending as far as about middle of third segment of antennular peduncle, about a third of the length of carapace. Carapace without dorso-median carina, rounded on antero-lateral angle. Supra-corneal spine absent. Extra-corneal spine well developed. Infra-corneal one absent (Fig. 6, *A*).

Abdomen a little more than twice the length of carapace, has no dorso-median carina. Sixth pleonal somite a little less than a third of the length of carapace. Telson sub-rectangular in upper aspect, more or less diminishing width posteriorly, ca. 0.4 times as long as carapace, does not reach to tip of uropods, three times as long as wide at posterior end, armed with two anterior and posterior pairs of spinules near lateral margin of dorsal surface at middle and at three-fourths respectively, furnished with a short and a long spinules on distal border close each lateral angle in addition to very long hairs developed on the same border (Fig. 6, *B*).

Antennular peduncle consists of three segments, first segment with a

large acute lateral process reaching to about middle of third joint of this peduncle, second segment about half the length of first one, third one as long as second one. Inner antennular flagellum somewhat shorter than carapace, divided into two inner longer and outer shorter rami at fourth segment; shorter ramus made up of five segments, provided with long hairs on ventral surface (Fig. 6, *C*). Outer antennular flagellum more or less longer than inner one. Antennal scale extending well beyond distal end of antennular peduncle, has an acute disto-lateral spine which is well exceeded by distal edge of the scale.

Mandible closely resembles that of *Athanas parvus* De MAN in general feature having two-jointed endopodite and incisor process armed with 7 rather large and 8 minute teeth on cutting edge (Fig. 6, *D*). Maxillula cross-shaped, endopodite has an apical lobe (Fig. 6, *E*). Maxilla shows close resemblance to that of *Athanas parvus* De MAN and *A. lamerifer* KUBO, endopodite small, unsegmented (Fig. 6, *F*). Third maxilliped pediform, with exopodite; antepenultimate joint about six times as long as wide, a little more than thrice the length of penultimate joint; ultimate one twice the length of penultimate one (Fig. 6, *H*).

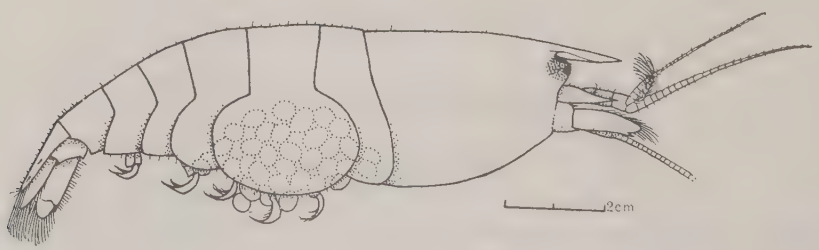

Fig. 5. *Atanas sctoensis* sp. nov. Ovigerous female, 8.6 mm in body-length

First pereiopods subequal in length, about two times as long as third maxilliped; merus ca. 1.6 times as long as ischium, about five times as long as wide at middle; carpus distally somewhat widens, about four times as long as wide at distal extremity, nearly equals to merus in length; chela about 0.8 times the length of carpus, palm twice as long as wide at middle, movable finger ca. 0.66 times as long as palm, cutting edges of both fingers simple (Fig. 6, *I*, *J*). Second leg as long as first one, somewhat slenderer than first one; merus ca. five times as long as wide; carpus five-segmented, one and half times the length of merus; chela small, a third the length of carpus, palm about 1.4 times as long as wide, as long as movable finger (Fig. 6, *K*). Fourth

leg more or less shorter than second leg; ischium 1, merus 1.8, propodus 1.9, dactylus 0.6 in proportion to carpus in lenth; merus about four times as long as wide; propodus ca. 8.5 times as long as wide, armed with equidistantly set short bristles along posterior margin; dactylus simple, sharply pointed at tip (Fig. 6, *L*). Fifth one closely resembles fourth one in general configuration (Fig. 6, *M*).

Eggs ovoidal, measure 0.40–0.45 mm in longer diameter, 0.28–0.30 mm in shorter one.

Type specimen: ovigerous female, 8.6 mm in body-length, collected, by Dr. T. Tokioka of Seto Marine Biological Station of Kyoto University, in the vicinity of the Station in Sept. 1949.

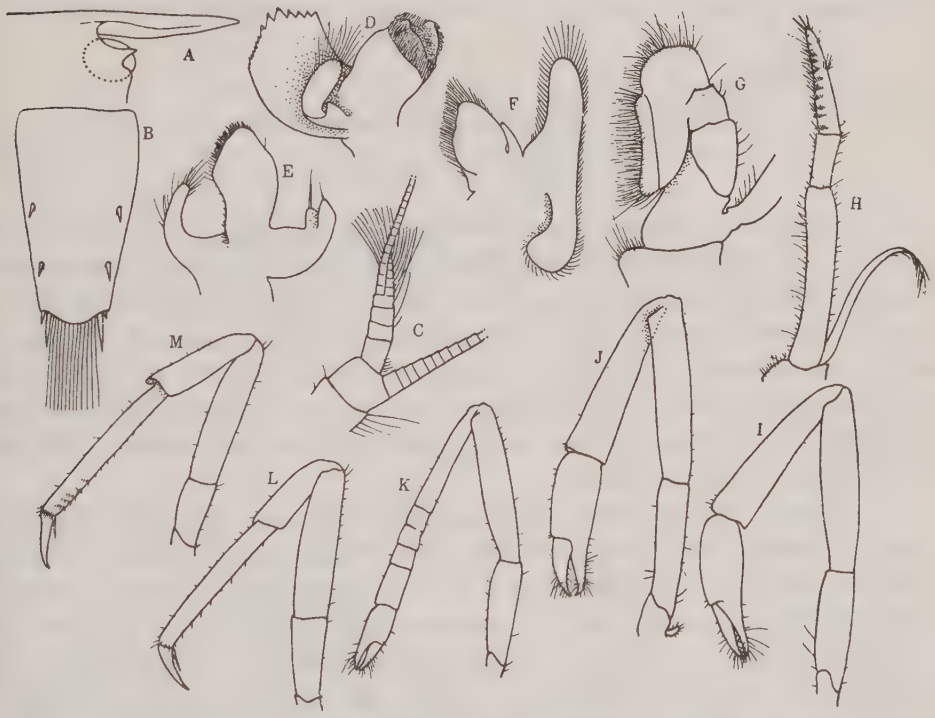

Fig. 6. Bodily parts of *Athanas setoensis* sp. nov. *A*, rostrum, eye, and extra-corneal spine, × 18; *B*, telson, × 26; *C*, antennule, × 18; *D*, mandible, × 40, *E*, maxillula, × 40; *F*, maxilla, × 26; *G*, second maxilliped, × 40; *H*, third maxilliped, × 18.5; *I*, right first cheliped, × 18.5; *J*, left first cheliped, × 18.5; *K*—*M*, second, fourth, and fifth legs respectively, × 18.5.

The species newly established herewith shows much affinity to such many species as *Athanas orientalis* Pearson, *A. esakii* Kubo, *A. haswelli* Cout., *A. amazone* Holthuis, *A. minikoensis* Cout., *A. sibogae* De Man, *A. jedanensis*

De MAN, *A. granli* COUT., *A. dimorphus* ORT., *A. dimorphus monoceros* HELLER, *A. lamellifer* KUBO, *A. parvus* De MAN, *A. kominatoensis* KUBO, *A. japonicus* KUBO, and *A. oshimai* YOKOYA in having no supracorneal spine, and to the following species, viz., *A. esakii*, *A. lamellifer*, *A. japonicus*, *A. oshimai*, *A. crosslandi* TATTERSAL, *A. orientalis*, *A. haswelli*, *A. minikoensis*, *A. dimorphus*, *A. dimorphus monoceros*, *A. areteformis*, COUT., *A. naifaroensis* COUT., *A. tenuipes*, *A. amazone*, *A. nitescens* (LEACH) [=*A. veloculus* BATE] in possession of posterior three pairs of pereiopods with simple dactylus. Also it has relation to *A. jedanensis* and *A. areteformis* in the character of carapace without infracorneal spine.

But the present new form is easily assortable from all the forms given above and others in the peculiarity of the first chelipeds in which the carpus is elongated, as long as merus, and considerably longer than the chela.

Periclimenes (Ancylocaris) tosaensis sp. nov.

(Figs. 7, 8, Tab. 2)

Shell smooth. Rostrum considerably ascendent, somewhat shorter than carapace, reaching a little beyond distal end of second segment of antennular peduncle, sharply pointed at tip, moderately deep, about six times as long as deep at deepest portion, dorsally convex with 7 teeth, ventrally also convex with 2 denticles near tip (Fig. 7). Carapace dorsally rounded, about a quarter of body-length, armed with epigastric, antennal, and hepatic spines, without supra-orbital spine; infra-orbital angle produced into an acuminate process; antero-lateral angle rounded.

Abdomen dorsally rounded in anterior two and last three (including telson) segments, but rather markedly carinated dorsally in posterior half of third segment and somewhat so in about anterior half of fourth one; fifth segment slightly less than half of carapace; sixth segment a little more than twice the length of fifth one; pleura of pleonic segments from first to sixth nearly straight on lower margin. Telson dorsally rounded, falls short of uropods, ca. four-fifths as long as carapace; dorsally provided with two anterior and posterior pairs of spinules, anterior pair stands a little backwards to middle of the length of telson, posterior one at midway between the first pair and the tip of telson (Fig. 8, *A*); posterior margin somewhat drawn out backwards, minutely pointed at middle, armed with three pairs of spinules (Fig. 8, *B*).

Eye extending beyond middle of rostrum.

Antennular peduncle made up of three segments; basal segment twice

as long as wide, furnished with a large basal lateral and a minute disto-lateral spines, distal border remarkably produced into a terminal lobe which is rounded on apex, extending beyond middle of second antennular segment; second segment one and half times as long as wide. somewhat narrower than half the width of basal segment; third segment as long as and as wide as second segment (Fig. 8, *C*). Outer antennular flagellum about 1.8 times the length of carapace, divided into outer longer and inner shorter branches at 14th joint, outer branch consists of 31 joints and the other one 10 joints. Inner antennular flagellum simple, about 1.3 times as long as carapace, composed of 37 joints (Fig. 8, *D*). Antennal scale four times as long as wide at base of disto-lateral spine; broadly rounded on anterior margin which reaches well beyond tip of disto-lateral spine (Fig. 8, *E*). Carpocerite extending to a third of antennal scale.

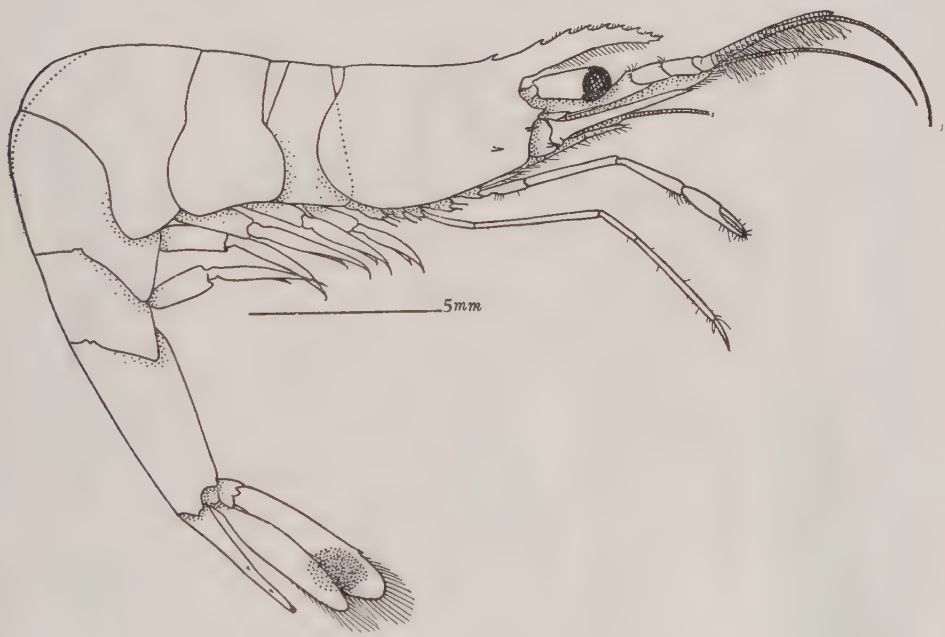

Fig. 7. *Periclimenes* (*Ancylocaris*) *tosaensis* sp. nov., male, 24.0 mm in body-length.

Mandible Y-shaped, without palp; molar process with five teeth irregular in size; incisor one armed with three teeth on cutting edge (Fig. 8, *F*). Maxillula cross-shaped; inner lacinia moderate in size, outer lacinia little longer than inner one, furnished with a number of setae on distal edge; endopodite unsegmented, with small palp (Fig. 8, *G*). Maxilla with an inner endite which has two narrow apical lobes; endopodite not jointed (Fig. 8, *H*). All maxillipeds carry an exopodite. First maxilliped with unsegmented pointed

small endopodite, and epipodite (Fig. 8, *I*). Second maxilliped J-shaped (Fig. 8, *J*). Third one pediform, extending as far as carpocerite; penultimate segment about six times as long as wide at middle, a little shorter than antepenultimate segment; last segment three-fourths the length of penultimate one (Fig. 8, *K*).

First cheliped reaching beyond tip of antennular peduncle by entire finger; merus ca. 8.5 times as long as wide, about 1.35 times as long as carpus; carpus dislally more or less increases in size, five times as long as wide at distal end; palm ca. 2.2 times as long as wide at middle; finger simple on cutting edge, a little shorter than palm (Fig. 7). Third leg slender, extending beyond tip of antennular peduncle by dactylus and a quarter of propodus, ischium 1.1, merus 2.6, propodus 2.4, dactylus 0.8 in proportion to carpus in length; dactylus simple (Fig. 8, *L*).

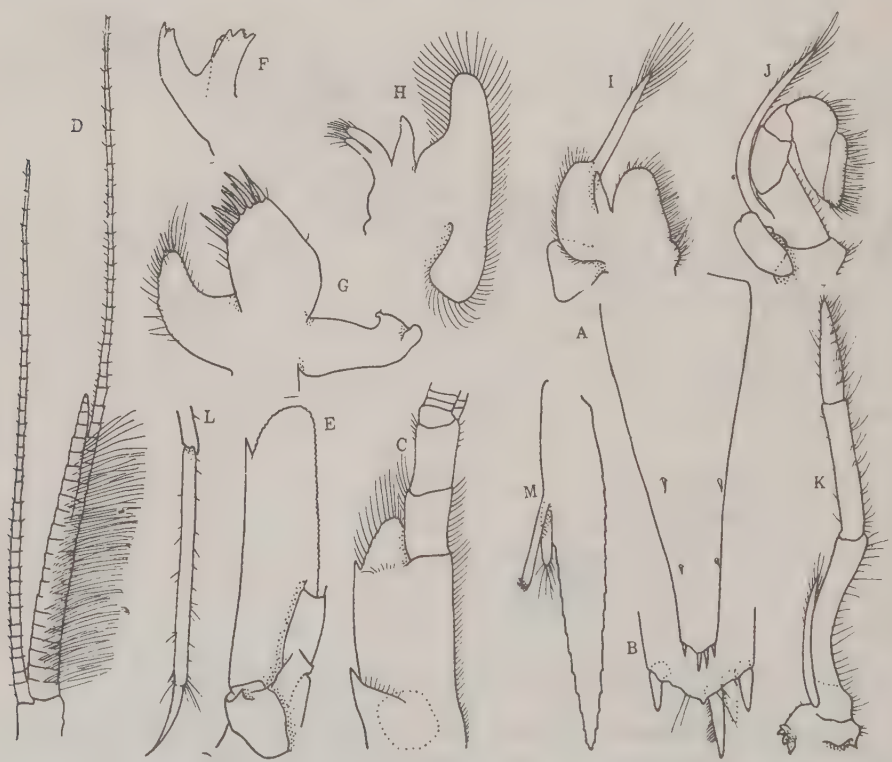

Fig. 8. Bodily parts of *Periclimenes* (*Ancylocaris*) *tosaensis*, sp. nov. *A*, upper aspect of telson, × 13; *B*, tip of telson, × 40; *C*, antennular peduncle, × 10; *D*, antennular flagella, × 10; *E*, antennal scale and carpocerite, × 9; *F*, mandible, × 18.5; *G*, maxillula, × 40; *H*, maxilla, × 18.5; *I*, first maxilliped, × 15; *J*, second maxilliped, × 18.5; *K*, third maxilliped, × 13.5; *L*, last two segments of third pereiopod; *M*, endopodite of second pleopod, × 18.5.

Second pleopod carries an endopodite which has an appendix interna and an appendix masculina at a thisd of inner edge (Fig. 8, *M*).

Outer uropods marked with a large reddish brawn (in formalin) colour near tip.

Type-specimen: male, 24 mm in body-length, 5 mm in carapecial length, missing the antennular flagella, second, fourth, and fifth pereiopods, taken by a trawl net and collected by Mr. T. Kawazawa in March, 1951.

Type-locality: off Usa, Takaoka-gun, Kôchi Prefecture.

The present species newly erected in this place appears to be most closly related to the Californian species, *Periclimenes* (*Ancylocaris*) *lucasi* CHACE (1937) in general feature, but it is separable from the latter as shown in the Table 2.

Table 2. Showing differences between the present new species and *P.* (*Ancylocaris*) *lucasi*.

Items	*P. tosaensis*	*P. lucasi*
(1) Rostrum	armed with 7 teeth on dorsal edge and tow ones on ventral rim; ventral edge more or less concave.	armed with 8–11 teeth on upper rin and 1–3 ones on lower rim; ventral edge straight or even slightly concave.
(2) Third pleonic somite	dorsally with rather developed hamp.	dorsally with strikingly developed hamp.
(3) Sixth pleonic segment	somewhat more than twice as long as fifth pleonic somite.	twice as long as fifth abdominal somite.
(4) Telson	much shorter (0.7 times as long as sixth pleonic somite.	a little longer than sixth abdominal segment.
(5) Outer antennular flagella	bifid at the 14th joint; inner ramus with 10 joints.	bifid at the 10th joint; inner ramus with four joints.
(6) Antennal scale	four times as long as wide at base of disto-lateral spine.	three times as long as wide at base of disto-lateral spine.
(7) Maxilla	inner endite distally orks into two lobes.	inner endite simple.

Ogyrides striaticauda KEMP

(Japanese name: Moyô-tsunome)

(Figs. 9, 10; Tab. 3)

Ogyrides striaticauda KEMP, 1915, pp. 284–289, figs. 28, a, b; 29, a–f; 30, a–g.

Apart from some minute points, the specimens dealt with tally well with the description and figures given by KEMP in all important traits. The

discrepancies between them are slight as shown in the Table 3, and seem to be only fluctuations within a species. Therefore they are identified to the present species.

Table 3. Differences between the present specimens and the Kemp's description of *Ogyrides striaticauda*.

Items	Present specimens	KEMP's description
(1) Rostral spines	8–10, mostly 8–9 in number.	7–9 in number.
(2) Sixth pleonic somite	a little longer than fifth one.	about equal to fifth one in length.
(3) Telson	nearly equal to, or slightly shorter than sixth pleonic somite, minutely bifulcate or abruptly truncate at tip.	a little longer than sixth pleonic somite, rather sharply angled at tip.
(4) Antennular flagella	different in length, about one and half times as long as antennular peduncle in inner ramus, and ca. 1.8 times as long as the peduncle in outer one.	are of the same length, about as long as antennular peduncle.
(5) Antennal scale	nearly reaches to distal end of the second segment of antennular peduncle; three times as long as wide at middle.	reaches only to the middle of the second segment of antennular peduncle; three and a quarter times* as long as wide.
(6) Fourth pereiopod	a little shorter than fifth pereiopod.	longest of the last three pairs of legs.

*, three times accoring to his figure.

Some supplements of the telson, rostral spines, antennule, second abdominal appendages, and eggs for the description worked out by KEMP, and additional illustrations may be given herein on basis of five males and four females.

Rostral spines run 8 to 10 in number. Of 9 specimens available, four have 9, three have 8, and two have 10 spines.

Telson as long as or slightly shorter than sixth abdominal somite, minutely bifurcate (Fg. 10, *D*) or abruptly truncate (Fig. 10, *E*) at distal extremity, furnished with peculiar striations on ventral surface as pointed aut by KEMP (Fig. 10, *C*).

Inner antennular flagellum thin, about 0.8 times as long as carapace, composed of 26 joints. Outer one much thicker than the other one in proximal about half, but as thin as the inner one in distal half, made up of 17 joints in proximal thicker portion and 16 joints in distal thinner part (Fig. 10, *F*).

Second pleopod well developed; basipodite a little more than twice, and about three and half times as long as wide at middle in male and female respectively; exopodite three times in male, 3.6 times in female as long as wide

at middle, provided on inner border with stylamblys at a position somewhat nearer to proximal end than to tip in male, conversely a littel nearer to distal end than to proximal one in female; stylamblys a third the length of endopodite in male, a quarter as long as endopodite in female (Fig. 10, *G*, *H*).

Egg small, globular, about 0.3 mm in diameter.

The description and illustrations given above are based on five males and four females, two of which are ovigerous, ranging 10.2–11.6 mm in body-length. The specimens secured from the Inland Sea (Kasaoka, Okayama Prefecture) by Mr. J. YASUDA in July, 1951.

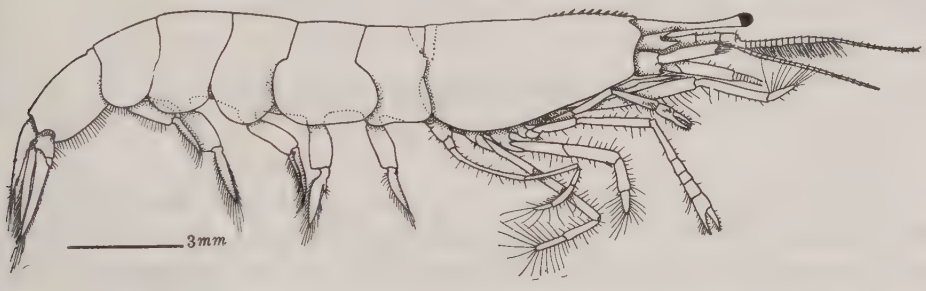

Fig. 9. Lateral aspect of a male of *Ogyrides striaticauda* KEMP.

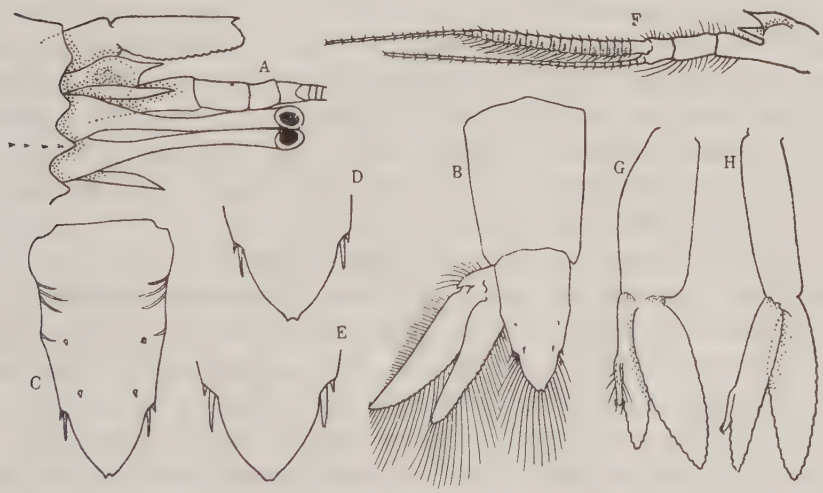

Fig. 10. Bodily parts of *Ogyrides striaticauda*. *A*. eyes, antennular peduncle, antennal scale, and others, × 18.5; *B*, sixth pleonic somite, telson, and uropods, × 15; *C*, telson showing the peculiar striations, × 26; *D*, distal part of telson of a male showing bifurcate tip, × 40; *E*, distal part of telson of a male showing abruptly truncate tip, × 40; *F*, antennule, × 15; *G*, second pleopod of a male, 10.5 mm in body-length, × 26; *H*, second pleopod of a female, 11.6 mm long, × 26.

Distribution: Chilka lake, Ennur, and Cochin near Ernakulan, India (KEMP).

Remarks: Accoring to YASUDA's information, this small shrimps begin to appear in shallow water in the neighbourhood of Kasaoka from early summer, and the specimens examined were taken by young-fish net.

Both sexes closely resemble each other in general appearance. Remarkable sexual dimorphism is, however, found to exist in the second abdominal appendages These appendages are markedly slenderer in female than in male as described already in addition to that presence or absence of the appendix masculina on the endopodite of the second pleopod.

Spirontocaris crassirostris sp. nov.

(Figs. 11, 12)

Rostrum roughly subcircular in profile, a little less than one and half times as long as wide at the widest region, extending as far as tip of antennular peduncle, furnished with well defined but low lateral ridge, armed with three pointed teeth on upper border and on apical border with seven similar teeth which are irregular in size; first tooth of the upper border inserted near base of rostrum, second tooth at about middle between base and tip of rostrum, and third one at three-quarters (Fig. 11). Carapace provided with dorso-median carina, three supraorbital, an antennal, and a pterygostomian spines. Dorso-median carina armed on anterior half with two acuminate spines, gradually diminishes in hight behind the last tooth. Of the three supra-orbital spines, two situated on orbital edge and one a little behind the two ones. Infraorbital angle spiniform.

Abdomen without dorso-median carina, about 3.3 times as long as carapace, each pleuron of anterior five pleonic segments furnished with six, eight, five, and four sharply pointed spines respectively on lower margin; the spines of the first pleonic somite minute. Sixth pleonic somite about half the length of carapace, carries postero-lateral angle which is spiniform, directing backwards, and an acute spine on pleuron near postero-lateral angle. Telson 2.25 times as long as wide at the proximal broadest portion or a little less than seven times as long as wide at distal end, about three times the length of sixth abdominal somite, armed with four pairs of spinules on dorsal surface; disto-lateral angle minutely spiniform; distal margin abruptly pointed at middle, furnished with two pairs of short and long spinules (Fig. 12, *A. B*).

Eye with well pigmented large cornea which is associated with an ocellus on postero-lateral edge; stalk short, as long as wide at proximal end,

about 0.6 times as long as cornea, armed with two inner and outer apines on distal dorso-median edge, outer one of the spines slender, sharply pointed at tip, directed forwards, the other one very swollen, with abruptly pointed tip which is directed medially (Fig. 12, *C*).

Antennular peduncle stout, nearly reaches to tip of rostrum; basal joint a little more than twice as long as wide at anterior end, has on basal outer edge a large acuninate process which is nearly reaching to distal end of second segment; intermediate joint as long as wide at proximal extremity, 0.4 times as long as basal joint, provided on outer rim with a large acute spine directing antero-laterally; last joint as long as intermediate one, carries a slender spine on dorsal surface near distal rim. Antennular flagella rather short; inner flagellum thin, consists of 14 joints, 0.7 times as long as peduncle on which it lies; outer flagellum strikingly swollen in proximal three quarters, made up of 20 joints, as long as peduncle (Fig. 12, *D*). Antennal scale large,

Fig. 11. *Spirontocaris crassirostris* sp. nov., male.

roughly triangular in shape, twice as long as wide at the widest portion, straight on outer margine, extends well beyond tip of antennular peduncle, bearing a large acut disto-lateral spine which stretches well beyond apical margin of lamella (Fig. 12, *E*). Carpocerite three times as long as wide, reaches to middle of second antennular peduncle.

Mandible made up of molar and incisor processes and palp; molar process has a large tooth on inner edge; incisor one carries four denticles on

cutting edge; palp made up of two segments, distal segment somewhat longer than basal one (Fig. 12, *F*).

Maxillula consists of inner and outer laciniae and endopodite; inner lacinia rather small, strongly recurved forwards; endopodte as large as inner lacinia, with a small apical lobe (Fig. 12, *G*). maxilla has two proximal and distal endites; proximal endite small, produced into a minute pointed lobule on disto-median angle; distal one large, distally divided into two lobes;

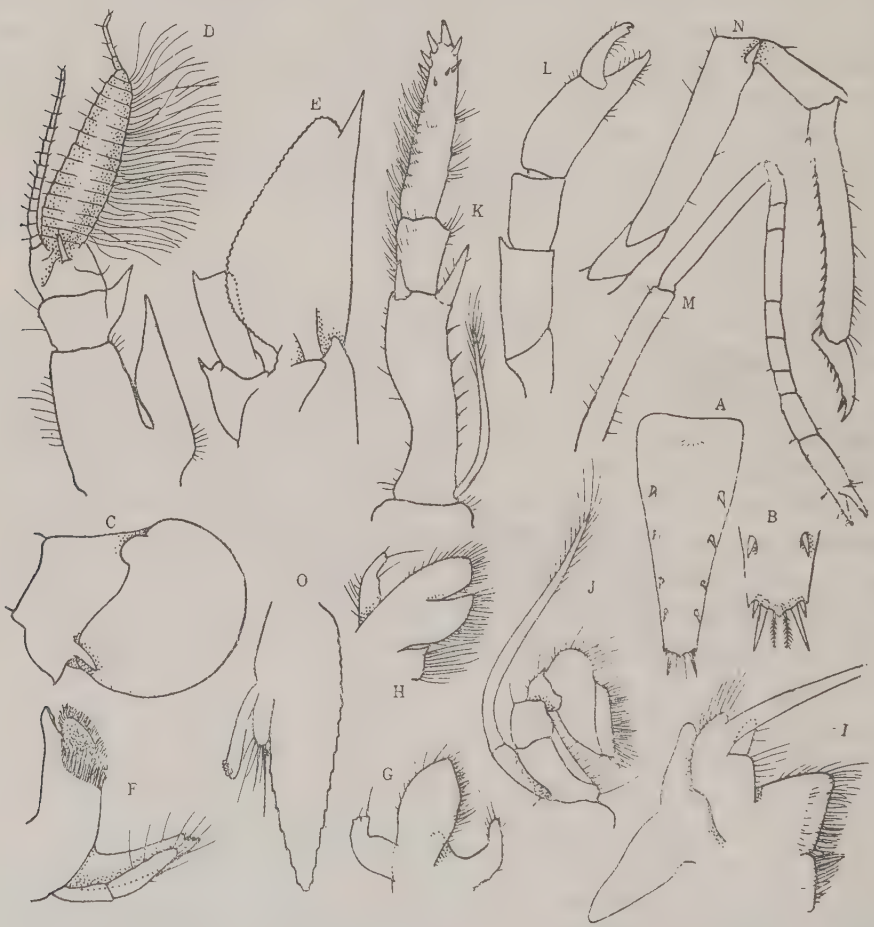

Fig. 12. Bodily parts of *Spirontocaris crassirostris* sp. nov. *A*, upper aspect of telson, × 10.5; *B*, distal part of telson; *C*, upper aspect of eye, × 18.5; *D*, dorsal view of antennule, × 13.5; *E*, antennal scale and carpocerite, × 13.5; *F*, mandible, × 26; *G*, maxillula, × 18.5; *H*, maixlla without exopodite, × 18.5; *I*, first maxilliped; × 26; *J*, second maxilliped, × 18.5; *K*, third maxilliped, × 13.5; *L*, first cheliped, × 13.5; *M*, second cheliped, 13.5; *N*, fourth pereiopod, × 13.5; *O*, endopodite of second pleopod, × 26.

endopodite not segmented (Fig. 12, *H*). All maxillipeds have well developed exopodite. First maxilliped with two segmented endopodite and bilobed epipodite; basal outer lobe of exopodite vestigial (Fig. 12, *I*). Second maxilliped ordinary in shape (Fig. 12, *J*). Third maxilliped stout, reaches to tip of antennal scale; antepenultimate segment three and half times as long as wide at distal end, armed with two spines on distal rim; penulimate segment a third the length of antepenultimate segment; ultimate segment about 2.3 times as long as penultimate segment (Fig. 12, *K*).

First cheliped stoutest of all legs, extending as far as tip of antennular peduncle; merus and palm subequal in length and width, about one and half times as long as carpus; movable finger equal to carpus in length, bifurcate at tip, simple on cutting edge as is the case with fixed one (Fig. 12, *L*). Second cheliped slender, reaching beyond tip of antennal scale by about half of chela; merus more or less shorter and thinner than ischipopdite; carpopodite 1.9 times as long as merus, seven-segmented; palm two and half times as long as wide, about 0.4 times the length of merus; fingers simple on cutting edges, ca. 0.7 times as long as palm (Fig. 12, *M*). Third pereiopod nearly stretching to tip of antennal scale; merus 2.5 times as long as ischium, five times as long as wide at middle, armed with a spine on distal outer edge; carpus subequal to ischium in length; propodus a little longer than merus, one-ninth as wide as long, with many equidistantly set spinules on posterior (or ventral) edge; dactylus as long as carpus, sharply pointed at tip, armed with several equidistantly set spinules on ventral margin. Fourth pereiopod reaches to the same level of tip of antennular peduncle, closely resembles third leg in general configuration, subequal to the third one in length (Fig. 12, *N*). Fifth leg similar to anterior two walking legs in general feature, somewhat shorter than fourth.

Endopodite of first pleopod bearing a stylamblys and an appendix masculina at a third of inner edge; the latter about half the length of the other one, provided with a number of long apical setae (Fig. 12, *O*).

Type-specimen: male, 26.5 mm in body-length (4.4 mm in carapecial length). The type specimen only is taken by a trawl net on November 11, 1949 at a depth of ca. 300 m off Heta, Izu peninsula.

The present new species is nearly related to *Spirontocaris pectinifera* (Stimpson) in general feature, especially in possession of many spines arranged on the lower margin of abdominal pleura, but it is at once sepatable from the latter by the shape of rostrum. Also the present new species bears resembrance to *Spirontocaris prionota* (Stimpson). However, it greatly differs from *S. prionota* in the following respects, viz. the rostrum is more circular

and having less number of spines developed on the apical border, and the abdominal pleura are armed with many spines on the lower border. The pleura are at all devoid of spines in *S. prionota.*

It may be mentioned herewith that the present species has much links to the genus *Thor* in the characteristics of the antennules. As described already, the outer antennular flagellum of the present species is strikingly swollen. This peculiarity is common to the genus *Thor*. Moreover the third joint of the antennular peduncle is provided with a well developed but slender spine on the dorsal surface near base of the outer antennular flagellum. In that place in *Thor* a movable plate is found to exist according to KEMP (1916, p. 387).

Genus **Lysmata, RISSO** 1826.

Lysmata, KEMP, 1914, p. 110; HOLTHUIS, 1947, pp. 18–19.
Hippolysmata, KEMP, 1914, pp. 112-113; HOLTHUIS; 1947, pp. 19–25.

The genera *Lysmata* and *Hippolysmata* have been distinguished by the only point that the upper antennular flagellum of the former is biramous and that of the latter uniramous. While these characters, according to the present investigation, are revealed to be no longer worthless for demarkation of those genera. Because it is found that the flagellum in question of *Lysmata* (*Hippolysmata*) *vittata* (STIMPSON) and *L.* (*Hippolysmata*) *kükenthali* which are formerly included within the genus *Hippolysmata,* is biramous although the short ramus is so minute as being hardly recognizable by naked eyes (Fig. 13, *D*). So that it may be said that the difference by which those two closely related genera have been assorted is not difference between two independent characters, and it is merely difference of degree within a trait. Basing on the reason mentioned above, the genus *Hippolysmata* should be included within the genus *Lysmata* in broad sense and it may be maintained as subgeneric gradation.

Lysmata (**Hippolysmata**) **kükenthali** (DE MAN)

(New Japanese name: Akamo-ebi)

(Figs. 13, A–M; 14. A–D; 15, Tabs. 4, 5)

Merhippolyte orientalis, De MAN, 1892, pp. 407–410.
Merhippolyte orientalis? De MAN 1902, p. 849, Pl. 26, fig. 56.

Hippolyte kükenthali, De MAN, 1902, p. 850.
Nauticaris unirecedens, PEARSON, 1905, p. 81.
Hippolysmata kükenthali, De MAN, 1907, p. 426; KEMP, 1914, p. 115–117, Pl. 6. fig, 11; BORRADAILE, 1917, p. 403; HOLTHUIS, 1947, p. 69–70.
Hippolysmata marleyi, STEBBING, 1919, p. 120, Pl. 18; 1921, p. 22.

Body smooth, without dorso-median carina.

Rostrum rather short, reaching to distal end of first segment of antennular peduncle or a littl beyond distal end of the segment, armed with 4 or 5 teeth, mostly 5 teeth on upper border. Of these teeth, usually hindmost one is always found just above post orbital margin. Lower border of rostrum distally provided with 1–3 teeth (1 tooth in most cases) (Tab. 4). Epigastric spine placed at about a quarter of carapace. Supra-orbital spine absent. Antennal spine present. Pterygostomian spine, if present, minute (Fig, 14, *A–C*).

Sixth abdominal segment about twice as long as fifth abdominal one, about a third the length of carapace. Telson about two and half times as long as wide at proximal widest portion, ca. twice as long as sixth pleonic somite, furnished on upper surface with two anterior and posterior pairs of spinules; those of anterior pair situated somewhat anterior to middle of telson, and the other pair at half way of distance between the first pair and tip of telson; distal margin pointed at middle, armed with inner and auter shorter spinules (Fig. 13, *A*, *B*).

Antennular peduncle three-jointed; first joint somewhat less than twice as long as wide at distal end, provided on outer basal margin with a large lanceolate process extending as far as distal end of the segment on which it lies; second joint as long as wide, half times as long as first joint; third joint subequal to second joint in length (Fig. 13, *C*). Antennular flagella long, about 1.3 times the length of body; upper flagellum more or less longer than the other, bifid into two dorsal and ventral branches at 24th joints (or at about one-fifth of the length), ventral branch minute, composed of only two joints. (Fig. 13, *D*).

Antennal scale extending beyond tip of antennular peduncle by about a quarter of its length, about thrice as long as wide at the widest region, with disto-lateral spine which reaches to distal margin of lamella (Fig. 13, *E*). Carpocerite reaches to middle of antennal scale.

Mandible without incisor process and endopodite, armed with three large teeth on masticatory edge (Fig. 13, *F*). Maxillula ordinary in shape; inner endite rather small; outer one rather large; endopodite with less developed apical lobe (Fig. 13, *G*). Maxilla furnished with proximal and distal laciniae

on inner base; proximal lacinia small and simple; distal one large in comparison to proximal lacinia, distally bifid; endopodite not segmented (Fig. 13, *H*). All maxillipeds with well develpod exopodite. First maxilliped made up of two inner broad laciniae, vestigial endopodite and exopodite; basal lateral lobe of exopodite rather broad, semi-circular in outline (Fig. 13, *I*). Second maxilliped normal in shape (Fig. 13, *J*). Third maxilliped pediform, extending beyond tip of antennal scale by about half of ultimate segment; antepenultimate and ultimate segments about 2.5 and 1.6 times as long as penultimate

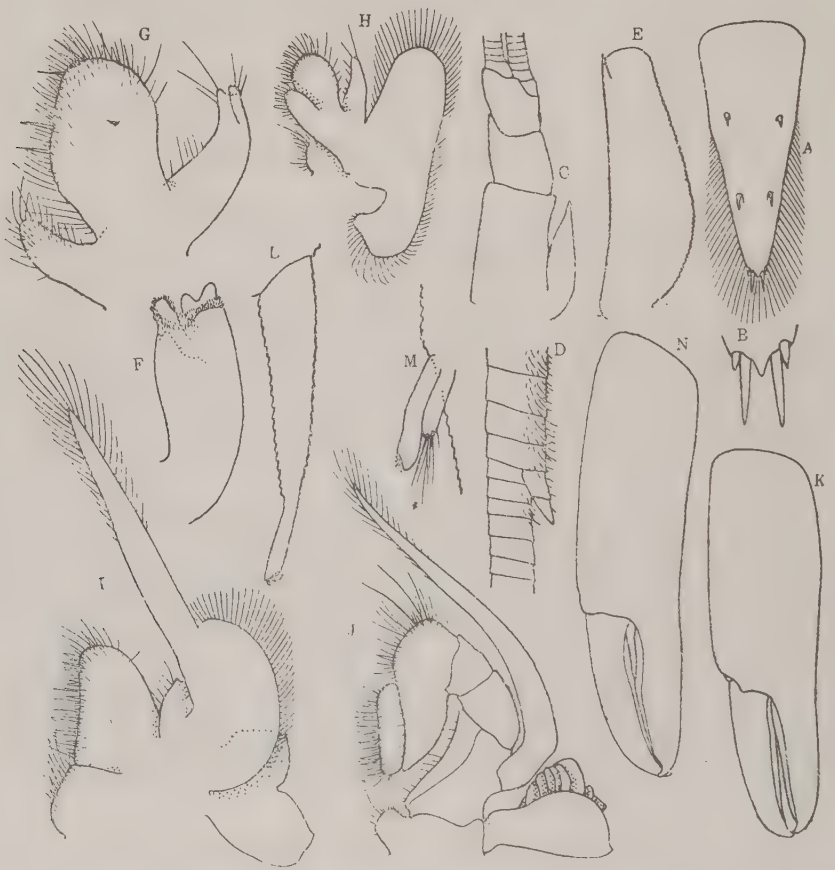

Fig. 13. Bodily parts of a male, *Lysmata* (*Hippolysmata*) *kükenthali* (*A—M*) and a female, *Lysmata* (*Hippolysmata*) *vittata* (*N*). *A*, upper aspect of telson, × 6.5; *B*, tip of telson, × 19; *C*, antennular peduncle, × 6.5; *D*, branched portion of upper antennular flagellum, × 19; *E*, antennal scale, × 6.5; *F*, mandible, × 19; *G*, maxillula, × 19; *H*, maxilla, × 9.5; *I*, first maxilliped, × 19; *J*, second maxiiliped, × 19; *K*, dorsal aspect of chela of first leg of female, × 13.5; *L*, endopodite of first pleopod, × 19; *M*, stylamblys and appendix masculina of endopodite of second pleopod, × 19; *N*, dorsal aspect of chela of second leg, × 13.5.

one respectively, exopodte of the appendage reaches to about middle of the segment on which it lies.

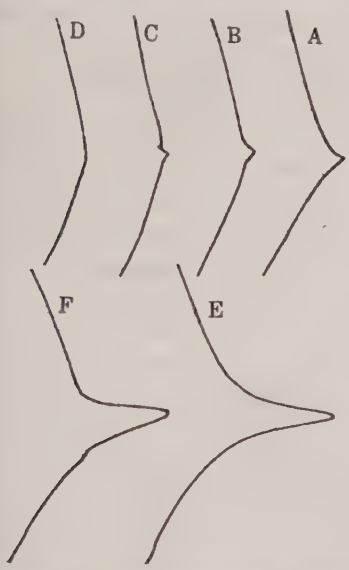

Fig. 14. Pterygostomian region of female of *Lysmata* (*Hippolysmata*) *kükenthali* (*A—D*) and *Lysmata* (*Hippolysmata*) *vittata* (*E, F*). Each figure, × 40. *A—F*, 31.0, 30.5, 44.0, 32.5, 37.0, and 40.3 mm in body-length respectively.

First cheliped reaches to tip of antennular peduncle, palm twice as long as wide, fingers about 0.7 times as long as palm, cutting edges when closed meeting throughout their length as mentioned by KEMP (1914, p. 116) (Fig. 13, *K*). Second chelipeds slender, subequal in length, surpassing tip of antennular peduncle by carpus and chela; carpus consists of 17–20 joints, usually 19 or 20 joints. Third leg extending beyond tip of antennular peduncle by last three segments; merus armed with one or two spinules on outer surface in distal a third; carpus about half as long as merus; propodus one and half times the length of carpus; dactylus about a quarter of propodus in length, provided on ventral border with four or five spinules which increase in size as they approach tward the tip. Last two legs subequal to third leg in feature and length.

Endopodite of first pleopod roughly elongated triangular in shape, has a large stylamblys on apex (Fig. 13, *M*).

Branchiae are arranged as shown in Table 5. A pleurobranchia present on last six thoracic segments. Arthrobranchiae entirely absent. A podobranchia exists on second maxilliped (Fig. 13, *J*). A mastigobranchia found on each thoracic appendage except last one; the mastigobranchiae developed on legs from first to fifth pairs are simply bar-shaped arising from anterior border of coxa of each pereiopod, and that atached on first maxilliped bilobed.

Ova small, about 0.4 mm in shorter diameter, 0.6 mm in longer diameter.

Colouration: Body furnished with broad bands of bright red colour. In carapace, those bands are five in number, and run obliquely. Of these five bands, the first one arises from branchiostegal region and runs along inferior and posterior margins of carapace. This band anteriorly extends to the bases of antennular and antennal peduncles and continues to ends of antennular peduncle and antennal scale. Second band runs from post-orbital margin to the region above the bases of third and fourth pereiopods. Third band stretches between the hindmost upper rostral spine and postero-lateral angle of

Table. 4. Measurement and counts of some items of *Lysmata* (*Hippolysmata*) *kükenthali* (De MAN) taken from Kominato, Chiba Prefecture. Unit of length, mm. *, spines found on carapace.

No.	Sex	Body-length	Carapace-length	No. of rostral spine Upper	Lower	Length of upper antennular flagellum
1	♀	44.0	10.5	3+2*=5	1	——
2	„	35.5	10.0	4+1 =5	1	44.0
3	„	32.5	9.5	4+1 =5	1	46.5
4	„	31.3	9.0	4+1 =5	1	41.5
5	„	31.0	8.7	3+1 =4	1	38.0
6	„	31.0	8.4	3+1 =4	1	39.0
7	„	31.0	8.9	4+1 =5	1	39.0
8	„	30.5	8.9	4+1 =5	1	39.0
9	„	29.0	8.4	3+1 =4	1	——
10	„	28.4	8.1	4+1 =5	1	——
11	„	27.4	8.3	4+1 =5	2	38.0
12	„	26.5	7.6	4+1 =5	1	31.0
13	„	25.2	7.0	4+1 =5	1	——
14	„	25.0	7.5	3+1 =4	1	——
15	♂	29.0	8.2	4+1 =5	3	45.5
16	„	28.0	7.9	3+1 =4	2	47.5
17	„	26.5	7.0	4+1 =5	3	——
18	„	23.5	6.1	4+1 =5	1	30.0

Table 5. Arrangement of the branchiae and exopodite of thoracic appendages of *Lysmata* (*Hippolysmata*) *kükenthali* (De MAN).

Branchiae	Thoracic segments I	II	III	IV	V	VI	VII	VIII	Total
Pleurob.	—	—	1	1	1	1	1	1	6
Arthrob.	—	—	—	—	—	—	—	—	0
Podob.	—	1	—	—	—	—	—	—	1
Mastigob.	1	1	1	1	1	1	1	—	7
Exopodite	1	1	1	—	—	—	—	—	3

carapace. Fourth one is nearly parallel to third band, arising from the base of epi-gastric spine. Fifth one lies starting from about middle between the epi-gastric spine and posterior margin of carapace. In abdomen, seven cross bands are found to exist. First band lies on first abdominal somite Second, third, fourth, and fifth bands cover posterior half of each somite from the

second to fifth respectively. Sixth band is placed on posterior part of sixth pleonic somite and anterior parts of telson and uropods. Last one lies on posterior half of telson and uropods. Third maxilliped, all pereiopods and pleopods are also stained with the same colour as shown in Fig. 15.

Sexual dimorphism: Sexual differences are found to exist in the respects given in the following lines. The body of male is somewhat slenderer than that of female. Upper antennular flagella of males are a little longer than those of females (Table 4). In addition to these, the endopodite of second pleopod demonstrates distinct difference between both sexes as that of male carries an appendix masculina which is absent in that of female.

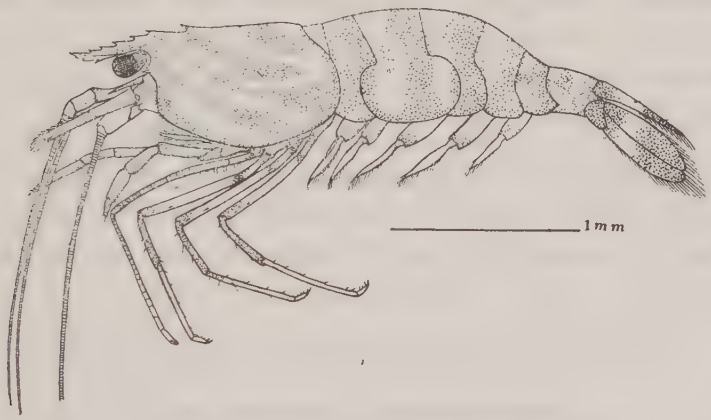

Fig. 15. *Lysmata* (*Hippolysmata*) *kükenthali* (De MAN), male, 29 mm long, showing colour pattern with dotting.

The description and figures given above are based on a male, 29 mm in body-length. Other three males, 23.5–27.5 mm long and 13 ovigerous females, 25.0–44.0 mm long were also examined. These specimens, except the largest female, were taken in September 10, 1947, by Dr. T. INO, Assistant Professor of our University, from Kominato, Chiba Prefecture in rocky shallow water at a depth of about 3 m. The largest female was securde in July 11, 1949, from the same locality.

Distribution: Isezela, Natal (STEBBING); Egmont Reef, Seychelles (BORRADAILE); Galle, and Jokkenpidi Paar, Ceylon (PEARSON); N. Cheval Paar, Ceylon (KEMP); Ternate, and Maoemere, Flores (De MAN); off Kawio and Kamboling Islands, Kawio group, N.E. of Celebes; off Atiatoening, west coast of New Guinea; South Lucipara Island; off Roemahloesi, northpoint of Tioor Island; Lamakera, Solor; Inhaca, Portuguese East Africa; Reunion Rocks, Isipingo near Durban (HOLTHUIS).

Remarks The pterygostomian spine is present or absent as shown in the Fig. 14, *A–D*. The spine is, if present, very minute, much smaller than that of *Lysmata (Hippolysmata) vittata*. It appears to be that there is a trend that the spine becomes smaller in size with the growth of this animal, and finally absent.

It should be mentioned herewith that the disagreements found between the present specimens and the description given by BORRADAILE and illustration made by KEMP are as follows. According to BORRADAILE's description (1917, p. 403), the carpus of the second pereiopod of Seychelles specimens is made up of 14 or 15 joints. While that of the present specimens consists of 17–20, mostly 19 or 20 ones. But the present specimens are in accordance with KEMP's description (1914, p. 116) of the same item of Ceylon specimens in which the wrist is composed of 19–21 joints.

The shape of fingers of the first pereiopod of the present specimens agrees well with the illustration given by KEMP (1914, Pl. 6, fig. 11), but the proportion of the finger to the palm in length is discrepant. The proportion of the present specimens is ca. 0.7 instead of being 0.5 in the KEMP's specimen. This difference may be, however, thought to be a fluctuation within a species, since this ratio is generally subject to considerable intra-specific variation.

Lysmata (Hippolysmata) vittata (STIMPSON)

(Japanese name: Akashimamo-ebi)

(Figs. 13, N; 14, E, F; 16; Tab. 6)

Hippolysmata vittata, STIMPSON, 1860, p. 26; LANCHESTER, 1901, p. 563; De MAN, 1907, pp. 423–426, pl. 33, figs. 49, 50; BALSS, 1914, p. 48; KEMP, 1914, pp. 113-115, pl. 6, figs. 6–10; 1925, p. 330; BORRADAILE, 1917, p. 403; HOLTHUIS, 1947, pp. 67–68.
Hippolysmata vittata var. ?, KEMP, 1914, p. 115.
Nauticaris unirecedens, BATE, 1888, p. 608, pl. 110, fig. 1.

Rostrum hardly reaches to distal end of second segment of antennular peduncle. Carapace with epigastric, antennal, and pterygostomian spines. Fifth abdominal segment about a quarter the length of carapace. Sixth pleonic somite one and half times as long as fifth pleonic one. Telson ca. 1. 6 times as long as sixth pleonic somite, armed with two pairs of spinules on dorsal surface and on distal edge respectively as is the case with *Lysmata (Hippolysmata) kükenthali*.

Antennular peduncle consists of three segments; first segment about three times as long as wide at distal end, carries a rather large acuminate

process which reaches to two-thirds of the segment on which it lies; second segment about half the length of first segment; third one a third as long as first segment. Upper antennular flagellum divided into two dorsal and ventral rami at 42nd joint or at a fifth of its length; lower ramus minute, hardly visible by naked eye, made up of only two joints. Antennal scale rather narrow, three and half times as long as wide at the widest region, reaches to tip of antennular peduncle, has a disto-lateral spine which extends as far as distal margin of lamella.

First cheliped surpassing distal end of antennular peduncle by major part of dactylus; merus four times as long as wide at middle; carpus about 0.6 times the length of merus; palm a little shorter than carpus; fingers 0. 6—0. 8 times as long as palm, meeting when closed in distal about two-thirds (Fig. 13, *N*). Second leg slender; ischium and merus subequal in length, merus composed of 9 joints; carpus about twice as long as merus, made up of 19–22 joints. Third leg extending beyond tip of antennular peduncle by distal half of carpus and last two segments; ischium 0.65, merus 2.1, propodus 1.1, dactylus 0. 28 in proportion to carpus in length; merus somewhat shorter than carapace, about 12 times as long as wide, armed with 4 spinules on distal half of outer surface along infero-lateral edge; dactylus carries 4 or 5 spinules on ventral margin. Fourth and fifth legs similar to third one in general configuration, more or less shorter than third one, stretching beyond tip of antennular

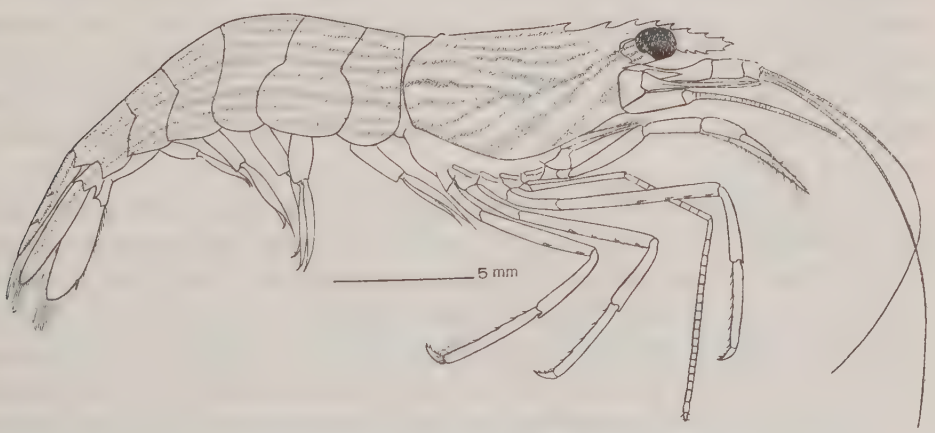

Fig. 16. *Lysmata* (*Hippolysmata*) *vittata* (STIMPSON) showing colour pattern, female, 29 mm in body-length.

peduncle by distal half of propodus and entire dactylus. Last leg equals to fourth one in length, having two or three meropoditic spines.

Colour pattern: body furnished with many red narrow longitudinal stripes as described by KEMP (Fig. 16).

The above mentioned description is based on a female, 29.0 mm in body-length. Besides 5 males ranging 17.5–28.0 mm in body-length, and 8 females, 21.5–40.5 mm long. were examined. Of those females, 4 ones, 29–40.5 mm long, were ovigerous.

Localities: Nagasaki, Nagasaki Pref. (Dr. S. MIYAKE); Tokushima (Mr. M. WATANABE); Tanabe Bay, Wakayama Pref. (Dr. UTINOMI); Miya, Aichi Pref. and Kominato, Chiba Pref. (I. KUBO).

Distribution: Hongkong (STIMPSON, BATE); Pulau Bidan, Penang (LANCHESTER); Madras; Kilakarai, South India; Ceylon; East Island, Andaman Islands; Karachi, Persian Gulf; Madras (KEMP); Liaotung Peninsula, N. China (YU); Inland Sea of Japan (De MAN); Red Sea; Ceylon; Amboina; Zushi, Sagami Bay, Japan (BALSS); Cargados Garajos; Seychelles (BORRADAILE); Lang Suen, Siam (Suvatti); Sebu, Philippines (THALLWITZ); Dirk Hartog Island, W. Australia (HALE); Krusadia and Shingle Islands (GRAVELY); Makassar; Koeandang Bay, N. Celebes; Woenoh Bay, northwestcoast of Waigeo; off Djedan Island, eastcoast of Aroe Islands; Kera near Timor; Amoy; Batavia (Fishmarket); Skroe, New Guinea; Balikpapan, Borneo (HOLTHUIS).

Remarks: The specimens examined well coincide with the descriptions and figures done by STIMPSON, KEMP, LANCHESTER, and others, except the matter given in annexed lines. As to the pterygostomian spine of the present species, KEMP (1914, p. 116) writes that "no spine at antero-lateral angle of carapace". In the present specimens, the pterygostomian spine is, however, present or absent (usually present) as described already, although it is minute when it is present.

The differences found between the present species and *Lysmata* (*Hippolysmata*) *kükenthali* (De MAN) which is a very close ally of the present species are as follows. In the former, the fingers of chelae of the first periopods, when closed, are meeting only in distal two-thirds, and the body is furnished with narrow longitudinal stripes of red colour, but in the latter, the fingers of the chelae are meeting throughout their length, and the body is provided with broad oblique and transverse bands as revealed already by KEMP. Besides these three things given in the Table 6 are found as differences between them.

Table 6. Differences between *Lysmata* (*Hippolysmata*) *vittata* and *Lysmata* (*Hippolysmata kükenthali*.

Items	*L. vittata*	*L. küenthali*
(1) Rostrum	more or less longer, hardly reaching to distal end of second segment of antennular peduncle.	short, usually hardly reaches to distal end of first joint of antennular peduncle.
(2) Upper antennular flagellum	divided into two rami at 42nd segment or at about a fifth of the length.	bifurcates at 23rd joint or at about one ninth of the length.
(3) Walking legs	with 4–5 spines on meropodite.	with a spine on meropodite.

REFERENCES

(Chief litteratures only are given below, and the papers marked with an asterisk were not avaiable to the writer)

ALCOCK, A. 1905. A revision of the genus *Peneus* with diagnosis of some new species and varieties. Ann. Mag. Nat. Hist., Ser. 7, vol. 16.

———. 1906. Catalogue of the Indian Decapod Crustacea in the collection of the Indian Museum, Part III, Macrura (Peneus). Calcutta.

ARMSTRONG, J. C. 1940. New species of Caridea from the Bermudas. Amer. Mus. Novit., No. 1096, pp. 1–10.

BALSS, H. 1914. Ostasiatische Decapoden II. Die Natantia und Reptantia. Abhand. math. -phys. Klasse K. Bayer. Akad. Wiss., II. Suppl.-Bd. 10 Abhandlg. München, pp. 101, Taf. 1.

———. 1924. Ostasiatische Decapoden V. Die Oxyrhynchen und Schlussteil. Archiv für Naturgeschichte, 90 Jahrgang, Abt. A, 5 Heft., pp. 19–84.

BATE, C. S. 1888. Report on the Crustacea Macrura collected by H. M. S. Challenger during the years 1873–'76. Rep. Voy. Challenger Zool. 24, pp. I–xc, 1–942, Text-figs. 1–76, Pls. 1–150.

BORRADAILE, L. A. 1917. On Carides from the Western Indian Ocean. Trans. Linn. Soc. London, pp. 397–412, Pls. 58–59.

BRASHNIKOW, V. 1907. Beiträge zur Fauna der russischen östlichen Meere, gesammelt von dem Schoner Storosch in den Jahren 1899–1902. Mémoires de l'académie impériale des sciences naturelles, Sér. 8, 20 (6), 1–185. (St. Petersburg).

BURKENROAD, M. D. 1934. The Penaeidae of Louisiana with a discussion of their world relationships. Bull. American Mus. Nat. Hist., 68 (2), 61–143.

CHACE, Fenner A., Jr. 1937. The Templeton Crocker Expedition. VII. Caridean Decapod Crustacea from the Gulf of California and the west coast of lower California. Zoologica, 22 (2), 109–138.

COUTIE'RE, H. 1905. Les Alpheidae in "the fauna and geography of the Maldive and Laccadive Archipelagos" edited by J. Stanley Gardiner, Cambridge, 2 (4), 852–918, Pls. 70–79.

HAAN, W. De. 1850. Fauna Japonica, Crustacea, xxxi,1–243, Tab. 1-55, A-Q.

HOLTHUIS, L. B. 1947. The decapoda of the Siboga-Expedition, Part 9. The Hippolytidae and Rhynchocinetidae. Siboga-Expeditie, Monographie, $39a_8$, 1–100.

———. 1949. The Caridean Crustacea of the Canary Islands. Zoologische Mededelingen, Deel 30, No. 15, 227–255.

———. 1951. The Caridean Crustacea of tropical west Africa. Atlantide Report, No. 2, 1–187.

KEMP, S. 1914. Notes on Crustacea Decapoda in the Indian Museum. 5. Hippolytidae. Rec. Ind. Mus., 10, 81–129, Pls. 1–7.

———. 1916. Notes on Crustacea Decapoda in the Indian Museum. 7. Further notes on Hippolytidae. Rec. Ind. Mus., 12, 385–405, Pl. 36.

———. 1922. Notes on Crustacea Decapoda in the Indian Museum. 15. Pontoniinae. Rec. Ind. Mus., 24, 113–288, Pls. 3–9.

———. 1925. Notes on Crustacea Decapoda in the Indian Museum. 17. On various Caridea. Rec. Ind. Mus., 27.

KOBJAKOVA, Z. I. 1936. Zoogeographical review of the decapoda from the Okhotsk and Japanese Seas. Travaux de la Société des Naturalistes de Leningrad, Sec. Zool., 65 (2), pp. 185–228.

KUBO, I. 1936. A description of a new alpheoid shrimp from Japan. Journ. Imp. Fish. Inst., 31 (2), 43–46.

———. 1936. Two new littoral Macrurous Crustaceans from Japan. Ibid. 31 (2), 47–54.

———. 1940. Studies on Japanese Palaemonoid shrimps, II. Pontoniinae. Ibid., 34 (1), 31–75.

———. 1940. On some littoral shrimps collected from Micronesia. Ibid., 34 (1), 77–99.

———. 1940. Notes on the Japanese shrimps of the genus *Athanas* with a description of one new species. Annot. Zool. Jap., 19 (2), 99–106.

———. 1942. On a new snapping shrimp, *Athanas kominatoensis*. Zool. Mag. (Tokyo), 54 (2), 82–85.

———. 1949. Studies on Penaeids of Japanese and its adjacent waters. Journ. Tokyo Coll. Fish. (Formerly Journ. Imp. Fish. Inst.), 36 (1), 1–467.

LANCHESTER, W. F. 1901. On the Crustacea collected during the "Skeat" Expedition to the Malay Peninsula, together with a note on the genus *Actaeopsis*. Proc. Zool. Soc. London, 534–574, Pls. 33, 34.

MAN, J. G. De. 1892. Decapoden des Indischen Archipels. Zoologische Ergebnisse Einer Reise in Niederländisch Ost-Indien, Bd. 2, 265–527, Tab. 15–29.

*———. 1902. Die von Herrn Prof. KÜKENTHAL in Indischen Archipel gesammelten Decapoden und Stomatopoden. In: KÜKENTHAL, W., Ergebnisse einer Zoologischen Forschungsreise in dem Molukken und Borneo. Abh. Senckenb. naturf. Ges., vol. 25, 467–929, Pls. 19–27.

———. 1907. On a collection of Curstacea, Decapoda and Stomatopoda, chiefly from the Inland sea of Japan; with descriptions of new species. Trans. Linn. Soc. London, 2nd Ser. Zool., 9 (2), 387–454, Pls. 31–33.

———. 1911. The Decapoda of the Siboga Expedition, Part 1. Monographie Penaidae. Siboga Expeditie, $39a_1$, pp. 1–131; Plates, I–X (1913).

———. 1911. The decapoda of the Siboga Expedition, Part 2. Alpheidae. Siboga-Expeditie, $39a_1$, pp. 133–465

NOUVEL, L. 1941. Contribution a l'étude systématique des *Athanas* des côtes de France. Bull. Inst. Oceanog. Monaco, No. 806, pp. 1–15.

ORTMANN, A. 1891. Die Decapoden-Krebse des Strassburger Museums. Zool. Jahrb., 5 Bd., pp. 437–542, Taf. 36, 37.

PEARSON, J. 1905. Report on the Macrura collected by Prof. HERDMAN, at Ceylon in

1902. HERDMAN's Report to the Government of Ceylon on the Pearl Oyster Fisheries of Manaar, Vol. 4, pp. 65-92, Pls 1-2.

PESTA, O. 1918. Die Decapodenfauna der Adria. Wien, pp. 1–500.

RATHBUN, Mary J. 1902. Japanese stalk-eyed Crustaceans. Proc. U.S. Nat. Mus. 26 (1307), 23–55.

———. 1902. The Brachyura and Macrura of Port Rico. Bull. U.S. Fish. Com. Vol. 20, Pt. 2, for 1900, pp. 1–127, Pls. 1–2.

———. 1906. The Brachyura and Macrura of the Hawaiian Islands. Bull. U.S. Fish Com., Vol. 23, for 1903, pp. 827–930, Pls. 1-24.

———. 1910. Crustaceans. Harriman Alsaka Expedition, 10 (Smithonian Institution Publication, 1997), pp. 1–190, Pls. 1-10.

———. 1929. Canadian Atlantic fauna. The Biological Board of Canada. The Atlantic Biological Station Publication, pp. 1–38.

SCHMITT, W. L. 1921. The marine Decapod Crustacea of California. Univ. Clifor. Pub. Zool., 23, pp. 1–470, Pls. 1–50.

———. 1933. Four new species of Decapod Crustaceans from Port Rico. Amer. Mus. Novi., No. 662, 1-9.

———. 1936. Zoologische Ergebnisse einer Reise nach Bonaire, Curacao und Aruba in Jahre 1930.Macruran and Anomuran Crustacea from Bonaire, Curacao and Aruba. Zool. Jahrb., 67 (5–6), 363–378, Pls. 11–13.

STEBBING, T. R. R. 1919. Some Crustacea of Natal. Ann. Durban Mus., vol. 3, 12–26, Pls. 1–5.

STIMPSON, W. 1860. Prodromus descriptionis animalium evertebratorum, quae in Expeditione ad Oceanum Pacificum Septentrionalem, a Republica Federate missa, C. RINGGOLD et J. R. DUCIBUS, observavit et descripsit. Pars 8. Crustacea Macrura. Proc. Acad. Nat. Sci. Philadelphia, pp. 22–48.

TATTERSAL, W. M. 1921. Report on the Stomatopoda and Macrurous Decapoda collected by Mr. C. CROSSLAND in the Sudanese Red. Sea. Jour. Linn. Soc. London, vol. 34, pp. 345–398.

URITA, T. 1942. Decapoda Crustaceans from Saghalien, Japan. Bull. Biogeog. Soc. Jap., 12 (1), 1–78.

YOKOYA, Y. 1930. Report of the biolgical survey of Mutsu Bay, 16. Macrura of Mutsu Bay Sci. Rep. Tôhoku Imp. Univ. Fourth Ser. Biol., 5 (3) 525–548, Pl. 16.

———. 1936. Some rare and new species of Decapod Crustaceans found in the vicinity of the Misaki Marine Biological Station. Jap. Journ., Zool., 7 (1), 129–146.